DOCUMENT RESUME

ED 219 232 SE 038 257

TITLE	PRISM Canada. Priorities in School Mathematics.
INSTITUTION	Alberta Univ., Edmonton. Faculty of Education.
REPORT NO	ISBN-0-88864-972-X
PUB DATE	Aug 81
NOTE	75p.
AVAILABLE FROM	Faculty of Education, University of Alberta, 4-116 Ed. N., University of Alberta, Edmonton, AB. T6G 2G5 Canada. $8. per copy.
EDRS PRICE	MF01 Plus Postage. PC Not Available from EDRS.
DESCRIPTORS	Curriculum Development; Curriculum Research; *Educational Needs; Educational Objectives; Educational Research; Elementary Secondary Education; *Mathematics Curriculum; *Mathematics Education; Mathematics Instruction; *Public Opinion; School Surveys; Surveys
IDENTIFIERS	*Canada; Mathematics Education Research; *Priorities in School Mathematics Project

ABSTRACT

The Priorities in School Mathematics (PRISM) Canada Project was initiated in September 1977 as a companion study to the National Council of Teachers of Mathematics (NCTM) PRISM Project. These twin projects were designed to help provide guidelines and suggestions for curriculum changes during the 1980's. Two instruments were used, a Preferences Survey and a Priorities Survey, and were given to both professional and lay samples. The view of professional preferences looked at content, goals, resources, methods, how much time should be spent on instruction, and calculators. Usually all of these were considered for the following content strands: 1) fractions and decimals; 2) algebra; 3) whole numbers; 4) geometry; 5) probability and statistics; 6) ratio, proportion and percent; 7) problem solving; 8) measurement; and 9) computer literacy. Lay opinion on preferences combines the content strands. The survey considered aspects of both the elementary and secondary curriculum and viewed issues such as student needs, content improvement, teacher education, upgrading activities, general problems, distribution of research funds, and methods of attacking problems. Implications for action highlights seven recommendations found in NCTM's "An Agenda for Action: Recommendations for School Mathematics of the 1980's." The document concludes with comments and criticisms about the survey made by respondents. (MP)

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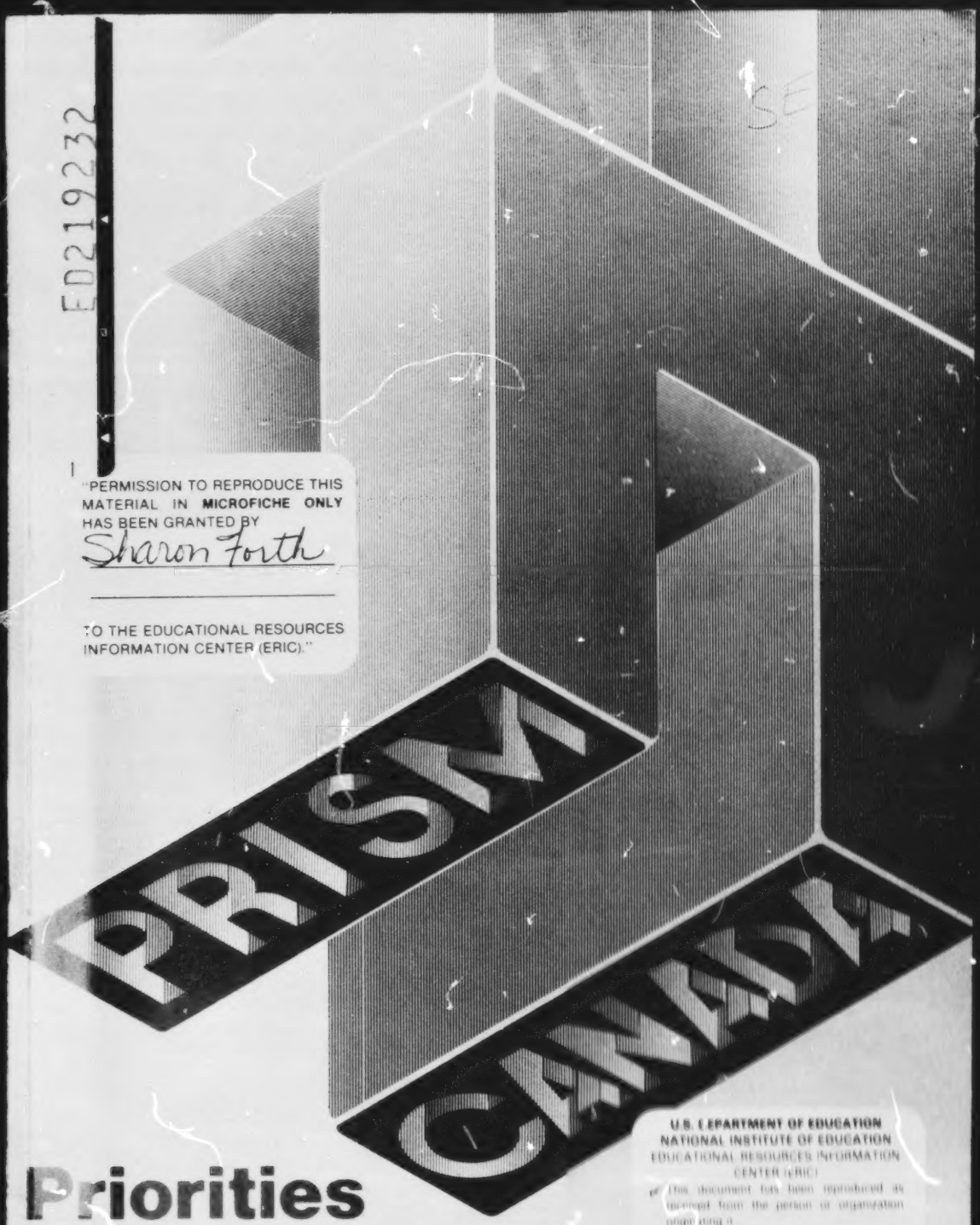

Priorities in School Mathematics

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First published by
The Faculty of Education,
University of Alberta
Edmonton, Alberta, Canada
1981

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PRISM (priorities in school mathematics)

"Companion study to the National Council of Teachers of Mathematics' PRISM project."
Bibliography: p.
ISBN 0-88864-972-X

1. Mathematics - Study and teaching.
2. Mathematics - Study and teaching - Canada.
3. Curriculum planning. I. Worth, Joan.
II. University of Alberta. Faculty of Education.
QA11.P74 510'.7'1 C81-091347-X

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Printed by:
University of Alberta Printing Services
Edmonton, Alberta, Canada

PRISM Canada

(Priorities in School Mathematics)

Supported by:

Alberta Advisory Committee for Educational Studies
Faculty of Education, University of Alberta
National Council of Teachers of Mathematics
University of Alberta Humanities and Social Sciences Research Fund

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August, 1981

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I. PURPOSE AND PLAN

The PRISM (Priorities in School Mathematics) Canada Project, initiated in September 1977 as a project of the Faculty of Education at the University of Alberta, was a companion study to the National Council of Teachers of Mathematics' PRISM Project. These twin projects were designed to help provide guidelines and suggestions for mathematics curriculum changes during the current decade. The findings are reflected in *An Agenda for Action* recently published by the National Council of Teachers of Mathematics (NCTM), which details recommendations for school mathematics of the 1980's.

The Canadian survey sample was drawn from ten groups of people interested in school mathematics; elementary teachers, secondary mathematics teachers, mathematics teacher educators, university mathematics teachers, mathematics supervisors, elementary school principals, secondary school principals, parents, school board members and employers.

To obtain a representative sample from each province, the proportion of the total Canadian population living in each province was calculated. Based on an anticipated sample of 1500 subjects, ten proportional subsamples were determined with the condition that there would be at least five subjects from each province in each category. The target sample distribution details are shown in Table 1.

Table 1
Canadian Sample

CATEGORY	NFLD. (2.4)*	P.E.I. (0.5)	N.S. (3.6)	N.B. (3.0)	QUE. (27.1)	ONT. (36.1)	MAN. (4.5)	SASK. (4.0)	ALTA. (7.8)	B.C. (10.8)	YUKON & N.W.T. (0.3)	TOTALS
PROFESSIONAL												
Teachers-Elem.	(4) **	(6)	(10)	(5)	(67)	(61)	(11)	(9)	(16)	(27)	0	(216)
	5	5	8	6	60	75	9	9	18	25		220
Teachers-Sec.	(5)	(5)	(10)	(5)	(60)	(82)	(11)	(9)	(18)	(28)	0	(230)
	5	5	8	6	60	75	9	9	18	25		220
Supervisors	(5)	(4)	(5)	(5)	(30)	(30)	(5)	(5)	(9)	(15)	0	(113)
	5	5	5	5	30	40	5	5	9	11		120
Mathematics-Educators	(5)	(1)	(5)	(4)	(34)	(33)	(5)	(5)	(9)	(15)	0	(112)
	5	5	5	5	30	40	5	5	9	11		120
Mathematicians	(5)	(5)	(5)	(5)	(30)	(30)	(7)	(5)	(8)	(9)	0	(109)
	5	5	5	5	30	40	5	5	9	11		120
Principals-Elem	(5)	(5)	(10)	(5)	(2)	(26)	(5)	(5)	(10)	(14)	0	(105)
	5	5	5	5	20	30	5	5	10	10		100
LAY	(5)	(5)	(9)	(5)	(10)	(11)	(6)	(5)	(10)	(14)	0	(80)
Principals-Sec.	5	5	5	5	20	30	5	5	10	10		100
Parents	(5)	(5)	(10)	(5)	(61)	(90)	(10)	(9)	(18)	(27)	0	(240)
	5	5	8	6	60	75	9	9	18	25		220
School Board Members	(5)	(5)	(5)	(5)	(30)	(34)	(6)	(5)	(9)	(11)	0	(114)
	5	5	5	5	30	40	5	5	9	11		120
Employers	(5)	(5)	(5)	(5)	(33)	(20)	(5)	(5)	(8)	(14)	0	(105)
	5	5	5	5	30	40	5	5	9	11		120
TOTALS	(49)	(46)	(74)	(49)	(375)	(417)	(71)	(62)	(114)	(170)	0	(1427)
	50	50	59	53	370	485	62	62	119	150		1460

* The percent of total Canadian population in each province.
Canadian population (1975) = 22,800,000. Source: Canada Handbook, 1977.

** Sample received

To obtain names and addresses of specific individuals a letter was sent to a Deputy Minister of Education in each province, outlining the purpose of the study and requesting assistance in identifying the persons to be surveyed. Each Deputy Minister arranged for the selection of the sample as he saw fit. The most common procedure was to request relevant organizations such as the teachers and trustees associations, universities, home and school associations, to supply the names. The sample identified from each province is also indicated in Table 1. All names submitted were used, giving an initial sample of 1427.

Two instruments were used in the study, a Preferences Survey and a Priorities Survey. The Preferences Survey presented a wide range of alternate content topics, instructional goals and methods and teaching resources for nine major areas (strands) in the K-12 mathematics curriculum. Survey respondents were asked to identify their level of preference for the alternatives to be incorporated in the school programs. The Priorities Survey asked respondents to judge the relative importance of various alternatives against each other. The items for the two instruments were developed jointly by the United States and Canadian project teams. In developing the instruments, the literature was reviewed to identify points of issue or concern, after which a working conference was held to develop an item pool. From this pool, trial instruments were developed, tested and subsequently revised. Demographic items were also included to collect data on the respondents. All items were machine scorable.

The item bank for the Preferences Survey contained nearly 800 items. Thus, an item sampling procedure was used so that any one subject answered only a portion of the items. This procedure resulted in several different forms being used for each group.

For the Priorities Survey, the sample was divided into two categories, professional and lay. The professional category included elementary teachers, secondary mathematics teachers, mathematics teacher educators, university mathematics teachers, mathematics supervisors and elementary school principals; the lay category included secondary school principals, parents, school board members and employers. Two different forms were used, one for the professional category and one for the lay category. Forms were randomly assigned to subjects.

The Preferences Survey was mailed to the total initial sample in March and April, 1979. The Priorities Survey was mailed to the same sample in May through July, 1979. If a Preferences Survey was returned because of an incorrect address or because the individual did not wish to participate, a Priorities Survey was not sent.

There were 761 (53.3%) Preferences Forms returned and 477 (34.2%) Priority Forms returned.

All returns were forwarded to the United States PRISM office at Ohio State University for analysis so that the Canadian and United States analyses were similar. Some special analyses were done on the Canadian data which would be of particular concern to Canadians.

A large amount of data was generated by this project and it is impossible to present or discuss it all in this report.

Also, several different analyses could have been done. For example, it would have been interesting to make regional comparisons but because of the item sampling techniques, there were too few regional responses to any given item to justify making comparisons.

We have chosen to compute and report percentages of both high and low preferences and priorities, by the professional and lay samples.

II. PREFERENCES SURVEY

The item sampling procedure resulted in twelve different forms being used for element ry teachers, secondary mathematics teachers, elementary school principals and university mathematics teachers; ten different forms for mathematics supervisors and teacher educators; and two different forms for secondary school principals, school board members and employers.

A slightly modified random assignment was used to assign the Preferences Forms to elementary and secondary teachers, elementary principals and mathematicians, because eight of the forms were designed to be answered by twice as many respondents as the other four.

Four of the forms for elementary and secondary teachers, elementary school principals and university mathematics teachers were translated into French along with one form for each of the other six groups. The French forms were sent to most of the Quebec sample and to any others who requested them.

Most of the items in the Preferences Forms came from nine content strands; fractions and decimals, algebra, whole numbers, geometry, probability and statistics, ratio and percent, problem solving, measurement, computer literacy.

Each strand contained items related to six specific topics; content, goals, resources, methods, who/time, and calculators. The typical item form for the Preferences Survey presented a range of alternatives related to the six specific topics for each strand. Respondents were asked to identify their level of agreement or preference for each of the alternatives.

Because of the large number of items and the sampling procedure used, the number of persons in any one group responding to a particular item was not large.

PROFESSIONAL OPINION

The professional sample which responded to the strand items included e'ementary teachers, secondary mathematics teachers, elementary principals, mathematics teacher educators, university mathematics teachers and mathematics supervisors.

Each of the nine strands is examined below. Items which were supported and not supported by the respondents are highlighted. Finally, more detailed data is provided in table form. The percentages in the "HIGH" column indicate the sum of the two most positive responses on a five-point scale, generally "agree completely" and "agree". Similarly, the "LOW" percentages represent the sum of the two most negative responses, "disagree" and "strongly disagree".

FRACTIONS AND DECIMALS

Content

Of the 10 items focused on elementary conten\, the following were given high preference:

- students should be taught to solve a division problem by first estimating whether the answer will be larger or smaller than the number being divided
- decimals should be developed as a means of naming numbers between numbers (e.g., 2.4 is between 2 and 3; 2.41 is between 2.4 and 2.5)
- least common multiple and greatest common divisor should be stressed as basic ideas related to fra. tions
- fractions should be presented as answers to division problems; for example, 7/12 means 7 ÷ 12
- operations with fractions should be taught only for fractions with small denominators (e.g., 12 or less)

The following items were given low preference:

- decimals should be introduced by relating them exclusively to money
- tables of common denominators (factors and multiples) should be given to students.

Table 2
Fractions and Decimals — Elementary Content

		HIGH	*LOW*
1.	All fractions should be written as decimals so that the operations then can be performed with a calculator	47.8	36.2
2.	Operations with fractions should be taught only for fractions with small denominators (e.g., 12 or less)	64.7	23.6
3.	Tables of common denominators (factors and multiples) should be given to students	31.9	53.6
4.	Students should use slide rules, graphs and charts (nomograhs) to solve problems involving fractions	44.9	36.2
5.	Least common multiple and greatest common divisor should be stressed as basic ideas related to fractions	74.3	12.8
6.	Students should be taught to solve a division problem by first estimating whether the answer will be larger or smaller than the number being divided	85.5	7.2
7.	Fractions should be presented as answers to division problems; for example, 7/12 means 7 ÷ 12	67.1	12.0
8.	Decimals should be introduced by relating them exclusively to money	11.4	61.4
9.	Decimals should be developed as a means of naming numbers between numbers (e.g., 2.4 is between 2 and 3; 2.41 is between 2.4 and 2.5)	82.6	4.3
10.	Fractions should be developed as measures of lengths	58.5	22.8

Among the ten secondary items, the following were accorded high preference:

- least common multiple and greatest common divisor should be stressed as basic ideas related to fractions
- decimals should be developed as a means of naming numbers between numbers (e.g., 2.4 is between 2 and 3; 2.41 is between 2.4 and 2.5)
- students should be taught to solve a division problem by first estimating whether the answer will be larger or smaller than the number being divided
- fractions should be presented as answers to division problems; for example, 7/12 means 7 ÷ 12

The following items received low preference ratings:

- decimals should be introduced by relating them exclusively to money
- tables of common denominators (factors and multiples) should be given to students

Table 3
Fractions and Decimals — Secondary Content

		HIGH	*LOW*
11.	All fractions should be written as decimals so that the operations on them can be performed with a calculator	43.9	45.3
12.	Operations with fractions should be taught only for fractions with small denominators (e.g., 12 or less)	40.4	43.9
13.	Tables of common denominators (factors and multiples) should be given to students	15.8	75.5
14.	Students should use slide rules, graphs and charts (nomographs) to solve problems involving fractions	29.9	49.2

15.	Least common multiple and greatest common divisor should be stressed as basic ideas related to fractions	80.7	12.3
16.	Students should be taught to solve a division problem by first estimating whether the answer will be larger or smaller than the number being divided	73.7	10.5
17.	Fractions should be presented as answers to division problems; for example, 7/12 means 7 ÷ 12	52.6	26.3
18.	Decimals should be introduced by relating them exclusively to money	6.5	80.7
19.	Decimals should be developed as a means of naming numbers between numbers (e.g., 2.4 is between 2 and 3; 2.41 is between 2.4 and 2.5)	76.8	12.5
20.	Fractions should be developed as measures of lengths	44.7	30.3

Goals

Six items were rated as important reasons for the inclusion of fractions and decimals in the elementary curriculum.

The first two of these are utilitarian:

- decimals are used in many vocations such as auto mechanics, carpentry, plumbing and so on
- consumers need decimals to compute "best buys"

The others are:

- determining how to add, subtract, multiply and divide common fractions illustrates basic mathematical processes and reasoning techniques
- decimals provide solutions to algebraic equations or number sentences
- fractions are interpreted as measurements; for example, 7/12 of a stick found by using a ruler
- common fractions are simple ways to illustrate division

Only two items were rated as definitely not reasons for including fractions and decimals in the curriculum:

- common fractions are a traditional part of the curriculum
- operations with decimals provide mental exercise

Table 4
Fractions and Decimals — Elementary Goals

		HIGH	*LOW*
21.	Determining how to add, subtract, multiply and divide common fractions illustrates basic mathematical processes and reasoning techniques	72.4	18.1
22.	Fractions are interpreted as measurements; for example, 7/12 is the length of a stick found by using a ruler	53.7	41.5
23.	Common fractions are a traditional part of the curriculum	15.8	77.9
24.	Decimals provide solutions to algebraic equations or number sentences	56.4	26.6
25.	Common fractions are used in college-level mathematics	40.4	41.5
26.	Decimals are used in many vocations such as auto mechanics, carpentry, plumbing and so on	90.4	6.5
27.	Common fractions are simple ways to illustrate division	52.2	29.8
28.	Operations with decimals provide mental exercise	29.5	56.8
29.	Rational numbers need to be contrasted to the sets of the whole numbers, integers and real numbers	46.3	37.9
30.	Consumers need decimals to compute "best buys"	64.9	26.6

Reasons for including fractions and decimals in the secondary curriculum were the following:

- the metric system of measurement uses decimals almost exclusively
- common fractions are used in many vocations such as auto mechanics, carpentry, plumbing and so on
- decimals are used in money
- determining how to add, subtract, multiply and divide decimals illustrates basic mathematical processes and reasoning techniques
- common fractions provide solutions to algebraic equations or number sentences; for example, 7/12 is the solution of $12x = 7$
- decimals are used in college-level mathematics

Only one item was considered not a reason for inclusion:

- decimals are a traditional part of the curriculum

Table 5
Fractions and Decimals — Secondary Goals

		HIGH	*LOW*
31.	Determining how to add, subtract, multiply and divide decimals illustrates basic mathematical processes and reasoning techniques	80.0	13.9
32.	The metric system of measurement uses decimals almost exclusively	87.7	10.8
33.	Decimals are a traditional part of the curriculum	29.2	58.4
34.	Common fractions provide solutions to algebraic equations or number sentences; for example, 7/12 is the solution of $12x = 7$	78.1	15.6
35.	Decimals are used in college-level mathematics	61.0	25.0
36.	Common fractions are used in many vocations such as auto mechanics, carpentry, plumbing and so on	85.7	14.3
37.	Decimals provide simple ways to illustrate division	48.4	27.5
38.	Operations with common fractions provide mental exercise	38.5	44.6
39.	Decimals are used in money	81.6	15.3
40.	Consumers need common fractions to compute "best buys"	47.7	32.3

Resources

The resources deemed most useful by at least 85 percent of the respondents were:

- resource booklets with applications of fractions and decimals
- student kits of measuring devices
- individual study materials for fractions and decimals
- drill and practice materials

Table 6
Fractions and Decimals — Resources

		HIGH	*LOW*
41.	A calculator designed so that fractions could be input and the answer would be displayed as a fraction	61.5	28.2
42.	Films and videotapes on fraction and decimal concepts	82.1	9.0
43.	Masters of worksheets and activities for fractions and decimals	79.7	12.7
44.	Individual study materials for fractions and decimals	91.0	1.3
45.	Student kits of measuring devices	92.3	0.0
46.	Manipulative materials such as fraction bars, strips, etc.	79.5	6.4
47.	Drill and practice materials	85.9	9.0

48.	Large-scale demonstration devices	79.7	3.8
49.	Resource booklets with applications of fractions and decimals	97.4	0.0
50.	"Magic response paper" to give immediate feedback by revealing the correct answer just after students have written their answers	79.5	12.8

Methods

Most methods items were given positive ratings with the top four being:

- operations with fractions are developed within the context of applications problems
- student worksheets are included for drill and practice on fractions and decimal topics at the conclusion of each lesson
- specific objectives, criterion-referenced testing and other materials are included to encourage use of a mastery learning or an individually paced model
- geometric pictures of physical models are used to represent computational algorithms for fractions and decimals

One item was viewed as having a negative influence:

- slower students are allowed to use calculators so they may keep up with the rest of the class

Table 7
Fractions and Decimals — Methods

		HIGH	*LOW*
51.	Geometric pictures of physical models are used to represent computational algorithms for fractions and decimals	66.7	4.4
52.	Operations with fractions are developed within the context of applications operations	57.8	23.4
53.	Operations with fractions are developed within the context of applications problems	86.9	4.4
54.	Student worksheets are included for drill and practice on fractions and decimal topics at the conclusion of each lesson	83.5	4.4
55.	Slower students are allowed to used calculators so they may keep up with the rest of the class	32.5	51.2
56.	Basic operations with fractions and decimals are developed through long-term student projects	39.6	31.9
57.	More than 50 percent of the instructional time is devoted to student use of individual study materials to develop and extend fraction and decimal use	52.8	19.8
58.	Basic fraction and decimal ideas are introduced through laboratory investigations	59.4	12.1
59.	Detailed notes are provided to guide the teacher in oral presentations of lessons on fractions and decimals	55.0	25.3
60.	Specific objectives, criterion-referenced testing and other materials are included to encourage use of a mastery learning or an individually paced model	70.4	8.8

Who/Time

Of the ten items in this category, there was high agreement on two:

- students should be taught fractions with small denominators useful in various vocations
- more attention should be given to operations with fractions than operations with decimals

Strong disagreement was evident with respect t. four items:

- fractions should be omitted from the curriculum
- only college-bound students should be taught fractions (e.g., in algebra courses)
- work with fractions should be delayed until seventh or eighth grade
- division of fractions should be omitted from the curriculum except for very bright children

Table 8
Fractions and Decimals — Who/Time

		HIGH	*LOW*
61.	More attention should be given to operations with decimals than operations with fractions	44.1	52.0
62.	More attention should be given to operations with fractions than operations with decimals	64.7	24.5
63.	Division of fractions should be omitted from the curriculum except for very bright children	27.5	61.8
64.	Work with fractions should be delayed until seventh or eighth grade	26.7	66.4
65.	Operations with decimals should be introduced before operations with fractions	47.1	28.4
66.	All students should master operations with decimals, but not all should be expected to master operations with fractions	39.6	52.4
67.	Operations with decimals should be included in the first or second grade mathematics program (the earlier the better)	27.8	53.4
68.	Fractions should be omitted from the curriculum	7.8	88.2
69.	Only college-bound students should be taught fractions (e.g., in algebra courses)	8.8	88.3
70.	Students should be taught fractions with small denominators useful in various vocations	79.4	16.6

Calculators

The use of calculators was considered appropriate when:

- finding the area of a lot whose length is 73.28 units and whose width is 35.92 units
- developing ideas about decimals
- homework involving problems with decimals
- reducing all quantities in a recipe by $\frac{1}{3}$

Calculator use was deemed inappropriate when:

- taking a test involving fractions
- homework involving problems with common fractions
- reducing fractions
- taking a test involving decimals

Table 9
Fractions and Decimals — Calculators

		HIGH	*LOW*
71.	Homework involving problems with common fractions	35.8	57.1
72.	Homework involving problems with decimals	62.3	29.0
73.	Developing ideas about common fractions	43.7	41.4

74.	Developing ideas about decimals	71.4	24.3
75.	Finding equivalent forms of a given fraction (i.e., 2/3 = 6/9 = 24/36)	42.9	40.0
76.	Reducing fractions	35.7	52.8
77.	Taking a test involving fractions	15.7	74.3
78.	Taking a test involving decimals	37.6	52.1
79.	Reducing all quantities in a recipe by ½	58.6	30.0
80.	Finding the area of a lot whose length is 73.28 units and whose width is 35.92 units	88.6	7.2

ALGEBRA

Content

Over 60 percent of the respondents thought the following topics in algebra warranted inclusion in the elementary curriculum:

- solving open number sentences
- writing equations to solve word problems
- making generalizations about numerical patterns
- graphing of number sentences
- using exponents (including scientific notation)
- evaluating formulas
- study of simple mathematical functions or mappings

Table 10
Algebra — Elementary Content

		HIGH	*LOW*
81.	Solving open number sentences	93.1	4.7
82.	Evaluating formulas	63.6	13.7
83.	Operating with signed numbers	51.2	28.0
84.	Graphing of number sentences	70.4	11.3
85.	Using exponents (including scientific notation)	63.6	15.9
86.	Writing computer programs	20.4	45.5
87.	Multiplying expressions like (+3) x (-5)	38.6	43.2
88.	Making generalizations about numerical patterns	79.5	9.1
89.	Writing algebraic expressions	59.1	18.2
90.	Study of simple mathematical functions or mappings	63.6	36.3
91.	Writing equations to solve word problems	84.1	9.1
92.	Studying structural properties of number systems (e.g., the commutative property)	52.2	27.3
93.	Using set notation	41.9	34.9
94.	Inequalities	56.8	15.9
95.	Studying finite systems (e.g., clock arithmetic)	36.4	27.3

Insofar as the secondary curriculum is concerned, over 75 percent of the respondents rated these eight topics worthy of attention:

- solving linear equations
- evaluating formulas
- writing equations to solve word problems
- work with signed numbers
- inequalities
- graphing of number sentences
- use of exponents (including scientific notation)
- study of simple mathematical functions or mappings

Table 11
Algebra — Secondary Content

		HIGH	*LOW*
96.	Solving linear equations	90.3	5.8
97.	Work with signed numbers	82.4	7.9
98.	Use of exponents (including scientific notation)	77.5	10.7
99.	Multiplying expressions like (a+3) x (b–5)	65.7	17.7
100.	Right-triangle trigonometry	64.0	24.0
101.	Writing equations to solve word problems	86.0	7.0
102.	Using quantifiers and set notation	29.7	37.6
103.	Studying finite systems (e.g., clock arithmetic)	49.5	29.7
104.	Solving systems of equations (e.g., two or more equations with two or more unknowns)	51.5	32.3
105.	Sequences and series	49.0	32.0
106.	Evaluating formulas	87.2	4.6
107.	Graphing of number sentences	77.7	12.9
108.	Writing computer programs	32.6	45.4
109.	Making generalizations about number patterns	57.5	24.1
110.	Study of simple mathematical functions or mappings	75.8	12.6
111.	Studying structural properties of number systems (e.g., the commutative property)	49.4	33.7
112.	Inequalities	79.3	9.1
113.	Proving algebraic generalizations	41.4	41.3
114.	Adding, subtracting, multiplying and dividing polynomial expressions	51.2	33.7
115.	Properties of classes of numbers (e.g., integers, rationals, reals)	59.8	29.9

For college-bound students not headed for a scientific career, over 60 percent of those responding favored four topics:

- probability functions (e.g., probability theory)
- trigonometric functions and their inverses
- exponential and logarithmic functions
- sequences and series

Over half thought the following topics should definitely not be included:

- algebraic structures (e.g., groups, rings, fields)
- the system of complex numbers
- limits and continuity
- approximating the roots to higher degree polynomial equations

Table 12
Algebra — College-Bound Students

		HIGH	*LOW*
116.	Matrix algebra (e.g., combinatorics)	54.9	32.2
117.	Finite mathematics (e.g., probability theory)	46.0	30.2
118.	Probability functions (e.g., probability theory)	71.4	19.1
119.	The system of complex numbers	31.7	55.6

120.	Trigonometric functions and their inverses	69.8	22.2
121.	Theory of equations (e.g., fundamental theorem, solvability)	54.9	35.5
122.	Analytic geometry (e.g., conic sections)	49.2	28.5
123.	Algebraic structures (e.g., groups, rings, fields)	12.7	73.0
124.	Categories of functions (e.g., algebraic exponential, transcendental)	33.3	46.0
125.	Introductory calculus	41.3	41.5
126.	Limits and continuity	37.1	53.2
127.	Sequences and series	60.4	22.2
128.	Exponential and logarithmic functions	63.5	20.6
129.	Approximating graphed data with best-fit polynomials	59.6	19.4
130.	Vectors and vector spaces	38.7	43.5
131.	Mathematical models	50.7	27.0
132.	Systems of non-linear equations	38.1	42.9
133.	Transformations applied to graphing	50.0	38.7
134.	Approximating the roots to higher degree polynomial equations	25.4	52.4
135.	Trigonometric identities and equations	53.2	25.8

Goals

The following goals for algebra were strongly endorsed:

- to learn how to apply mathematics
- to consolidate arithmetic skills
- to build the background for taking more mathematics
- to learn to read mathematics
- to gain an appreciation for a type of mathematics that is more powerful and versatile than arithmetic
- to gain skills necessary for work and vocations

Table 13
Algebra -- Goals

		HIGH	*LOW*
136.	To consolidate arithmetic skills	78.5	5.8
137.	To learn to read mathematics	73.5	4.0
138.	To preserve options with respect to career and vocational choice	51.0	9.8
139.	To assure adequate scientific manpower	42.1	18.6
140.	To gain skills necessary for work and vocations	70.6	8.8
141.	To gain an appreciation for a type of mathematics that is more powerful and versatile than arithmetic	72.5	11.8
142.	To prepare for college	60.7	16.6
143.	To build the background for taking more mathematics	77.5	8.8
144.	To understand the use and power of computers	63.7	10.8
145.	To learn how to apply mathematics	89.1	2.0

Resources

The resources rated most positively were:

- booklets of algebraic applications to contemporary problems
- masters of worksheets and activities
- physical materials and equipment for laboratory experiments
- booklets of games and recreational activities that can be analyzed algebraically

	Table 14 Algebra — Resources	*HIGH*	*LOW*
146.	Calculators that can display the equation of a line given the coordinates of two points	55.2	29.9
147.	Individual study carrels equipped with computer-assisted instruction terminals and videotape cartridge players	61.6	15.2
148.	Booklets of algebraic applications to contemporary problems	87.2	12.8
149.	Masters of worksheets and activities	81.4	18.6
150.	Physical materials and equipment for laboratory experiments	76.5	8.3
151.	Personal computers for every two students	57.0	22.1
152.	Booklets of games and recreational activities that can be analyzed algebraically	71.7	20.0
153.	Calculators that will display the roots of a linear or quadratic equation when the coefficients are input	37.6	37.7
154.	Materials with minimal reading requirements	47.7	26.7
155.	Computer-driven graphing and plotting equipment	44.7	28.2

Methods

Most items were given positive ratings with the top four being:

- student worksheets are included for drill and practice on algebraic topics at the conclusion of each lesson
- algebraic concepts are inferred from the general patterns of arithmetic
- geometric models, simple machines and other applications are used to develop algebraic concepts
- problems that arise in the social or natural sciences are used to extract and develop algebraic concepts

	Table 15 Algebra — Methods	*HIGH*	*LOW*
156.	Problems that arise in the social or natural sciences are used to extract and develop algebraic concepts	75.4	7.8
157.	Student worksheets are included for drill and practice on algebraic topics at the conclusion of each lesson	81.6	3.9
158.	Algebraic concepts are inferred from the general patterns of arithmetic	80.6	3.9
159.	Basic algebraic ideas are developed using computing devices	47.4	14.4
160.	Basic algebraic ideas are introduced through laboratory investigations	57.9	18.4
161.	Deductive sequences are used to develop new algebraic ideas and structural characteristics	59.8	10.4
162.	Geometric models, simple machines and other applications are used to develop algebraic concepts	77.7	9.2
163.	Ideas are introduced through long-term, realistic student projects	57.9	19.7
164.	More than fifty percent of the instructional time is devoted to student use of individual study materials to develop and extend algebraic ideas	56.6	15.8
165.	It is expected that students will read formal presentations of basic algebraic ideas before classroom activities are devoted to these ideas	37.7	35.1

Who/Time

On ten items concerning the sequencing of algebraic topics and their audience, strong agreement was expressed on three:

- different algebra courses should be offered for students with different interests and abilities
- a special algebra course for vocational students should be offered
- algebra should be combined with geometry and other mathematical areas instead of being taught in separate courses

There was strong disagreement with the following:

- formal work with algebra should be dropped from the school curriculum since it bears so little relation to real world problems
- trigonometry should not be included in algebra courses at any level
- algebra should be studied for two years before taking a course in geometry
- for many students, a "historical and cultural mathematics" course should be substituted for algebra

Table 16
Algebra — Who/Time

		HIGH	*LOW*
166.	Every student graduating from high school should be required to take a full-year algebra course	57.1	36.6
167.	Algebra should be studied for two years before taking a course in geometry	17.1	69.3
168.	A special algebra course for vocational students should be offered	72.4	9.9
169.	For many students, a "historical and cultural mathematics" course should be substituted for algebra	25.7	52.2
170.	The theme for algebra courses should be functions	33.4	32.1
171.	Algebra should be combined with geometry and other mathematical areas instead of being taught in separate courses	66.4	24.8
172.	Different algebra courses should be offered for students with different interests and abilities	82.3	10.6
173.	By 1990, the skills and concepts of the traditional beginning algebra course of the 1970's should be acquired before students enter ninth grade	32.4	39.6
174.	Trigonometry should not be included in algebra courses at any level	11.5	73.4
175.	Formal work with algebra should be dropped from the school curriculum since it bears so little relation to real world problems	14.2	80.5

Calculators

The use of calculators was considered most appropriate for:

- checking answers
- working with limits of sequences
- simplifying expressions containing irrational numbers

Table 17
Algebra — Calculators

		HIGH	*LOW*
176.	Taking an algebra test	49.4	44.0
177.	Making a graph from a given equation	49.3	30.7
178.	Finding the solution of an equation	56.0	30.7

179.	Finding the value of d, when $d = 1/2gt$, if $g = 32$ and $t = 5$	53.3	30.6
180.	Working algebra word problems	50.6	38.6
181.	Demonstrating that $(a + b) = a + b$ for several specific values of a and b	52.0	38.7
182.	Solving systems of linear equations (two or more equations with two or more unknowns)	45.3	37.3
183.	Checking answers	86.7	6.8
184.	Simplifying expressions containing irrational numbers	64.0	29.3
185.	Working with limits of sequences	68.0	16.0

WHOLE NUMBERS

Content

Several whole number topics received support for inclusion in elementary school mathematics:

- mental calculations without the aid of paper, pencil or calculator
- techniques of estimation
- mathematics puzzles and games
- developing operations simultaneously in order to emphasize relationships between them
- specific strategies for solving word problems
- specific consumer skills

Topics not supported for inclusion were:

- paper and pencil algorithm for calculating square root
- only the most efficient algorithm for each operation is taught

Table 18

	Whole Numbers — Elementary Content	*HIGH*	*LOW*
186.	Several different algorithms (methods) for each of the four basic operations so that children can choose the method they prefer	53.9	35.2
187.	Techniques of estimation	90.4	8.7
188.	Specific strategies for solving word problems	75.0	15.2
189.	Mathematics puzzles and games	82.8	17.3
190.	Operations with signed numbers or integers	60.9	27.2
191.	Justification of each step of an algorithm by relating it to basic number properties	39.1	42.4
192.	Mental calculations without the aid of paper and pencil or calculator	90.4	7.7
193.	Multiplication and division developed simultaneously to emphasize relationships between them	77.1	12.0
194.	Specific instructions for operating a four-function calculator	41.6	25.8
195.	Tests of divisibility	65.2	18.4
196.	Only the most efficient algorithm (method) for each operation is taught	36.9	46.8
197.	A paper and pencil algorithm (method) for calculating square roots	37.6	51.6
198.	Computational and/or checking shortcuts (eg., casting out nines)	67.4	23.9
199.	Addition and subtraction developed simultaneously to emphasize relationships between them	77.4	15.0
200.	Specific consumer skills like balancing a checkbook and calculating "best buys"	73.2	21.5

Goals

The goals for teaching whole number concepts and skills most strongly supported were:

- to acquire the skills necessary for consumer decisions
- to develop the fundamental understandings upon which other mathematics is built
- to develop logical thinking ability
- to understand the structure of mathematics

Clearly rejected as a goal was:

- to preserve a traditional emphasis in the curriculum

Table 19
Whole Numbers — Goals

		HIGH	*LOW*
201.	To acquire the qualifications necessary for obtaining many jobs	70.0	12.2
202.	To be able to do well on standardized tests	38.5	35.2
203.	To understand the structure of mathematics	77.0	11.0
204.	To develop logical thinking ability	80.3	8.8
205.	To gain an appreciation for the beauty of numbers	60.5	15.4
206.	To develop disciplined work habits	72.6	9.9
207.	To acquire the skills necessary for consumer decisions	84.7	6.6
208.	To develop the fundamental understandings upon which other mathematics learning is built	83.3	7.8
209.	To preserve a traditional emphasis in the curriculum	18.7	39.6
210.	To learn to read mathematics	71.6	9.1

Resources

When presented with ten different resources for teaching whole number concepts and skills, the majority of respondents indicated they would either definitely want the resource, or the resource would be nice to have.

Resources listed were:

- resource books compiling examples of arithmetic applied to real-life situations
- standardized practice tests for basic skills
- masters of worksheets and activities
- physical materials for each student to use in modeling basic operations and algorithms
- short videotapes to illustrate basic computational algorithms
- packages of materials for individual student study
- a calculator for every student
- "magic response paper" to give immediate feedback by revealing the correct answer just after students have written their answers
- small programmable calculators or computers
- audiotapes for verbal drill and practice

Table 20
Whole Numbers — Resources

		HIGH	*LOW*
211.	A calculator for every student	69.7	21.1
212.	Short videotapes to illustrate basic computational algorithms	77.6	11.8
213.	"Magic response paper" to give immediate feedback by revealing the correct answer just after students have written their answers	69.7	18.5

214.	Audiotapes for verbal drill and practice	65.8	13.1
215.	Physical materials for each student to use in modeling basic operations and algorithms	81.6	6.5
216.	Masters of worksheets and activities	84.0	9.3
217.	Resource books compiling examples of arithmetic applied to real-life situations	96.0	0.0
218.	Small programmable calculators or computers	68.4	18.4
219.	Packages of materials for individual student study	77.3	5.3
220.	Standardized practice tests for basic skills	92.1	3.9

Methods

Asked to express positive or negative feelings about ten different teaching strategies, the respondents were positive about the use of materials as indicated in three items:

- manipulative materials are used in a mathematics laboratory at least once a week
- physical materials such as rods and area blocks are given to every student to use to model whole number algorithms and generate answers
- geometric pictures are used as models for computation

The use of calculators produced a generally negative response, with over half the respondents indicating negative feelings about:

- calculators are used instead of teaching paper and pencil algorithms
- slower students are allowed to use calculators in order to keep up with the rest of the class

Correspondingly, the feelings were positive about:

- the introduction of calculators is postponed until students have learned both the meaning of the operations and the paper and pencil algorithms for them

Table 21
Whole Numbers — Methods

		HIGH	*LOW*
221.	Geometric pictures are used as models for computation	67.8	10.2
222.	Calculators are used instead of teaching paper and pencil algorithms	25.4	50.8
223.	More than fifty percent of the instructional time is devoted to drill and practice when teaching the basic facts	61.0	23.7
224.	The introduction of calculators is postponed until students have learned both the meaning of the operations and the paper and pencil algorithms for them	84.7	1.7
225.	Physical materials, such as rods and area blocks, are given to every student to use to model whole number algorithms and generate answers	76.3	8.5
226.	Slower students are allowed to use calculators in order to keep up with the rest of the class	32.8	51.8
227.	Specific objectives, criterion-referenced testing and other materials are included to encourage use of a mastery learning or an individually paced model	69.0	6.9
228.	Manipulative materials are used in a mathematics laboratory at least once a week	86.4	3.4
229.	Activities are included which require going outside the classroom (perhaps on field trips) so that whole number operations may be illustrated with real-life examples	62.7	15.3
230.	More than fifty percent of the instructional time is devoted to student use of individual study materials to develop and extend whole number ideas	52.6	27.1

Who/Time

Regarding to whom and when whole number topics should be taught, the respondents strongly agreed with this statement:

- every student should master whole number computations with paper and pencil before graduating from high school

Somewhat contradictorily, respondents tended to agree that:

- students who cannot master paper and pencil computations by the end of grade eight should be required to take a special ninth grade mathematics course on the use of hand-held calculators.

The respondents did not agree with:

- leaving remedial work with whole number computations until adult school or junior college
- college-bound students spending three weeks every school year reviewing whole number computations
- not introducing algorithms for multi-digit computations until grade seven

Table 22
Whole Numbers — Who/Time

		HIGH	*LOW*
231.	Every student should master whole number computations with paper and pencil before graduating from high school	92.4	5.1
232.	College-bound high school students should spend at least three weeks of every school year reviewing whole number computation	37.1	53.9
233.	Students who cannot master paper and pencil computations by the end of grade eight should be required to take a special ninth grade mathematics course on the use of hand-held calculators	53.1	31.7
234.	The appropriate time to do remedial work with whole number computation is in an adult school or junior college after students can see the need and importance for calculating	10.1	79.8
235.	Algorithms for multi-digit computations should not be introduced until grade seven when students are more mature	30.4	53.2

Calculators

Strong support was expressed for using calculators for:

- checking answers
- doing a chain of calculations involving several different operations
- solving word problems
- adding the cost of several items in a grocery cart
- doing homework
- finding the divisors of a given number

Equally strong opposition was expressed toward using calculators when:

- multiplying 3 x 13
- taking a test on whole number computation
- calculating change from a five dollar bill

Table 23
Whole Numbers — Calculators

		HIGH	*LOW*
236.	Learning basic number facts	45.1	47.1
237.	Doing homework	68.0	24.0
238.	Taking a test on whole number computation	19.6	76.5
239.	Learning properties of different operations	56.8	31.4
240.	Doing the division 641 ÷ 17	56.9	37.2
241.	Multiplying 782 x 59	59.6	34.6
242.	Solving word problems	71.1	23.1
243.	Subtracting 2,150–1.983	49.0	49.0
244.	Multiplying 3 x 13	14.0	86.0
245.	Checking answers	86.2	9.8
246.	Calculating change from a five dollar bill	28.8	67.3
247.	Doing a chain of calculations involving several different operations	76.5	17.6
248.	Learning why the long division algorithm works	59.7	21.2
249.	Adding the cost of several items in a grocery cart	71.2	27.0
250.	Finding the divisors of a given number	65.4	21.1

GEOMETRY

Content

Topics receiving the strongest support for inclusion at the elementary level were:

- geometry of symmetry
- properties of triangles and rectangles
- parallel and perpendicular lines
- similar figures
- coordinate geometry
- three dimensional geometry
- congruence by transformations
- geometry of tesselations

Topics which should definitely not be included in elementary school were:

- logical reasoning principles including axioms and proofs
- congruent (matching) figures by the methods of Euclid

Table 24
Geometry — Elementary Content

		HIGH	*LOW*
251.	Geometry of tesselations (tiling)	59.4	16.2
252.	Geometry of distance and direction (vector geometry)	40.5	23.5
253.	The geometry of shadows (projective geometry)	51.3	29.7
254.	Constructions with a straightedge and compass	54.0	37.8
255.	Similar figures (magnification and reduction)	70.2	13.5
256.	Congruence by transformations (slides, flips and turns with movement of figures to match)	62.1	16.2
257.	Logical reasoning principles including axioms and proofs	19.4	63.9
258.	Properties of triangles and rectangles	83.7	10.8

259.	Three-dimensional geometry	62.1	16.2
260.	Parallel and perpendicular lines	75.6	16.2
261.	Geometry on a sphere (globe)	36.1	30.6
262.	Congruent (matching) figures by the methods of Euclid	25.0	50.0
263.	Properties of circles	44.5	41.7
264.	Geometry of symmetry	86.5	2.7
265.	Coordinate geometry (associating number pairs with points)	66.7	19.5

Those items which definitely should be studied by all students at the secondary level were:

- similar figures (magnification and reduction)
- properties of triangles and rectangles
- coordinate geometry
- properties of circles
- congruence by transformations or reflections
- geometry of distance and direction (vector geometry)
- three-dimensional geometry
- geometry of the sphere (globe)

There was strong agreement that three topics should not be studied by all high school students:

- symbolic logic and truth tables
- non-Euclidean geometries
- finite geometries

Table 25
Geometry — Secondary Content

		HIGH	*LOW*
266.	Geometry of tesselations (tiling)	46.3	19.4
267.	Geometry of distance and direction (vector geometry)	79.1	16.4
268.	Geometry of shadows (projective geometry)	43.3	28.4
269.	Similar figures (magnification and reduction)	92.6	4.5
270.	Congruence by transformations (moving figures to match) or reflections	80.6	6.0
271.	Logical reasoning principles including axioms and proofs	58.2	31.4
272.	Properties of triangles and rectangles	92.6	3.0
273.	Three-dimensional geometry	65.7	20.9
274.	Geometry of the sphere (globe)	61.2	19.4
275.	Congruent (matching) figures by methods of Euclid	56.7	28.4
276.	Properties of circles	86.1	7.0
277.	Coordinate geometry (associating number pairs with points)	90.3	5.6
278.	Finite geometries (e.g., nine point geometry)	20.8	43.1
279.	Symbolic logic and truth tables	25.0	52.8
280.	Non-Euclidean geometries	29.1	47.2

There was very little agreement on which advanced geometry topics should be included for college-bound students who would not be science or mathematics majors. The three topics receiving most support were:

- coordinate (analytic) geometry
- straightedge and compass constructions
- vectors

Those receiving least support were:

- network theory
- transformations by matrices
- study of axiomatic structures
- symbolic logic and work with truth tables
- projective geometry
- finite geometries

Table 26
Geometry — College-Bound Students

		HIGH	*LOW*
281.	Locus theorems	43.2	28.4
282.	Straightedge and compass constructions	70.6	14.7
283.	Coordinate (analytic) geometry	88.3	4.4
284.	Symbolic logic and work with truth tables	35.3	48.5
285.	Vectors	61.8	22.0
286.	Transformational geometry	53.0	20.6
287.	A variety of proof formats (e.g., paragraph, flow diagram, two-column)	45.6	25.0
288.	Non-Euclidean geometry	40.3	38.8
289.	Transformations by matrices	22.0	50.0
290.	Finite geometries	22.4	40.3
291.	Projective geometry	20.6	42.7
292.	Geometry of the sphere	48.5	25.0
293.	Solid geometry	48.6	30.9
294.	Network theory	16.4	50.7
295.	Study of axiomatic structures	26.5	48.5

Goals

Respondents reacted positively to all of the choices presented as reasons for teaching geometry. The two overwhelmingly popular reasons were:

- to develop logical thinking abilities
- to develop spatial intuitions about the real world

The other eight receiving strong positive agreement were:

- to learn to read and interpret mathematical arguments
- to acquire the knowledge needed for study of more mathematics
- to develop skills and knowledge needed by the consumer
- to learn to make proofs
- to practice arithmetic and algebraic skills
- to develop job oriented skills
- to motivate students who dislike computation
- to appreciate historical and cultural development

Table 27
Geometry — Goals

		HIGH	*LOW*
296.	To motivate students who dislike computation	59.6	19.2
297.	To develop job oriented skills	60.6	11.2
298.	To appreciate historical and cultural development	48.5	18.2

299.	To learn to make proofs	62.7	10.1
300.	To develop spatial intuitions about the real world	91.9	0.0
301.	To learn to read and interpret mathematical arguments	79.6	5.1
302.	To practice arithmetic and algebraic skills	60.6	12.1
303.	To develop logical thinking abilities	94.9	1.0
304.	To develop skills and knowledge needed by the consumer	70.7	8.1
305.	To acquire the knowledge needed for study of more mathematics	78.6	6.1

Resources

All respondents indicated a desire to have nine of the ten given resources for teaching geometry:

- source books with applications of geometry to real problems
- short films or videotapes showing basic geometric concepts
- manipulative material and lab experiments
- a kit of measuring tools for each student
- large-scale demonstration models and devices
- individual study materials
- computer generated and animated graphics
- 35mm slides of basic geometric figures for blackboard projection
- masters of worksheets and activities

The resource least desired was:

- drafting tables and equipment

Table 28
Geometry — Resources

		HIGH	*LOW*
306.	Computer generated and animated graphics	80.0	11.6
307.	A kit of measuring tools for each student	85.0	13.4
308.	Short films or videotapes showing basic geometric concepts	88.3	3.3
309.	Masters of worksheets and activities	78.3	3.4
310.	Individual study materials	80.0	6.7
311.	Resource books with applications of geometry to real problems	88.4	3.4
312.	Drafting tables and equipment	46.6	35.0
313.	Large-scale demonstration models and devices	80.0	10.0
314.	35mm slides of basic geometric figures for blackboard projection	80.0	11.7
315.	Manipulative material and lab experiments	86.6	13.4

Methods

The four teaching strategies receiving the highest degree of support from all respondents were:

- basic geometry ideas are introduced through laboratory investigations
- activities requiring students to go outside the classroom to measure things
- student worksheets are included for drill and practice
- simulations where students play consumer/worker roles using geometry in real world situations

Other methods for teaching geometry receiving support were:

- long term projects designed to be assigned to individuals or to teams of students
- mastery learning, or an individually paced model are encouraged
- detailed notes are provided to guide the teacher in oral presentation of geometry lessons
- activities included that anticipate the class being divided into small discussion groups
- more than 50% of instructional time is devoted to student use of individual study materials

Table 29
Geometry — Methods

		HIGH	*LOW*
316.	Activities are included that would require students to go outside the classroom to measure things	73.5	14.1
317.	Students are expected to read formal presentations of basic geometric ideas before classroom activities are devoted to these ideas	39.4	39.5
318.	Student worksheets are included for drill and practice on geometric topics at the conclusion of each lesson	71.9	7.0
319.	Basic geometric ideas are introduced through laboratory investigations	74.7	2.8
320.	Specific objectives, criterion-referenced testing and other materials are included to encourage use of a mastery learning or an individually paced model	67.1	5.7
321.	More than fifty percent of the instructional time is devoted to student use of individual study materials to develop and extend geometric ideas	57.2	7.0
322.	Activities are included that anticipate the class being divided into small discussion groups	62.0	11.3
323.	Simulations where students play the role of consumer or worker using geometry in real world situations	71.9	11.3
324.	Detailed notes are provided to guide the teacher in oral presentation of geometry lessons	63.3	14.1
325.	Long term projects designed to be assigned to individuals or to teams of students	67.6	11.3

Who/Time

Respondents were asked to react to the placement of geometry topics at different points in the curriculum.

All groups expressed strong agreement that:

- a full-year course in applied geometry (navigation, measurement, intuitive development of concepts) should be available as a high school elective course.
- geometry modules should be developed for insertion in present mathematics courses or combined to form short-courses
- intuitive geometric concepts are at least as important in grade one as number concepts

There was strong disagreement expressed about two of the suggestions:

- no geometric topics should be taught before seventh grade
- a full year course in geometry should be delayed until students have taken two years of algebra

Table 30
Geometry — Who/Time

		HIGH	*LOW*
326.	A full year course in geometry should be delayed until students have taken two years of algebra	18.7	58.7
327.	A full-year course in applied geometry (navigation, measurement, intuitive development of concepts) should be available as a high school elective course	77.4	6.6
328.	No geometric topics should be taught before seventh grade	5.2	90.8
329.	Separate courses in geometry should be abolished and geometry content integrated with other mathematics in grades K-12	58.6	30.7
330.	Geometry modules should be developed that could be inserted in present mathematics courses or combined to form short-courses lasting from two to twelve weeks	64.5	17.1
331.	Geometric topics presently taught in elementary schools form an adequate minimum knowledge of this area for high school graduation	23.7	57.9
332.	A second year of advanced geometry should be offered in high schools	36.5	21.6
333.	Much of the mathematics taught in grade four should be geometry allowing a "pause" in the development of arithmetic concepts until children can reach a new developmental level	19.7	54.0
334.	Intuitive geometric concepts are at least as important in grade one as number concepts	63.1	21.1
335.	More of the mathematics curriculum in grades seven and eight should be devoted to geometry	36.1	23.7

Calculators

Respondents generally reacted favorably to the use of calculators when the geometry activity involved computation. In particular, the following activities were considered very appropriate:

- finding the length of the third side of a right triangle using the Pythogarean theorem
- finding the circumference of a circle, given the diameter
- using trigonometry to find the length of a side of a triangle
- computing the area of a trapezoid

The use of the calculator was considered not appropriate when:

- finding the measure of the complement or supplement of a 57 degree angle
- finding the midpoint of a line given the coordinates of the end points as (2,3) and (7,1)

Table 31
Geometry — Calculators

		HIGH	*LOW*
336.	Finding the midpoint of a line given the coordinates of the end points as (2,3) and (7,1)	27.1	55.9
337.	Computing the area of a trapezoid	69.5	23.7
338.	Finding the length of the third side of a right triangle using the Pythagorean theorem	81.4	11.9
339.	Find the measure of the complement or supplement of a 57 degree angle	32.2	57.6
340.	Doing geometry homework	50.9	30.5
341.	Finding the circumference of a circle given the diameter	76.3	13.6

342.	Calculating the volume of a cone, when the diameter of the base is six cm. and the height is ten cm.	52.5	33.9
343.	Using trigonometry to find the length of a side of a triangle	71.1	11.9
344.	Calculating the coordinates of the new vertices of a triangle after a given transformation	50.8	22.0
345.	Taking a geometry test	52.5	33.9

PROBABILITY AND STATISTICS

Content

Respondents thought the following topics should be included for elementary pupils:

- collection and organization of data (e.g., graphs, tables).
- Predicting outcomes
- reading and interpreting statistical information
- decision-making (e.g., for voting or consumer situations)
- measures of central tendency (e.g., mean, median, mode)

But the following ought not to be offered at the elementary level:

- calculating probabilities of compound and conditional events
- measures of spread (e.g., range, quartiles, etc.)
- combinations and permutations

Table 32
Probability and Statistics — Elementary Content

		HIGH	*LOW*
346.	Collection and organization of data (e.g., graphs, tables)	92.2	6.3
347.	Predicting outcomes	79.7	6.3
348.	Reading and interpreting statistical information	72.3	12.3
349.	Measures of central tendency (e.g., mean, median, mode)	52.2	34.4
350.	Measures of spread (e.g., range, quartiles, etc.)	21.5	52.3
351.	Calculating the probability of an event occurring	39.0	34.4
352.	Combinations and permutations	33.8	50.8
353.	Testing of conjectures and hypotheses	50.8	32.3
354.	Calculating probabilities of compound and conditional events	37.5	62.5
355.	Decision-making (e.g., for voting or consumer situations)	65.6	15.6

The following topics were chosen as most appropriate for college-bound students:

- calculating probabilities of compound and conditional events
- combinations and permutations
- probability distributions (e.g., normal, binomial)
- curve fitting and prediction
- correlation

Respondents accepted the following items as appropriate for all secondary students:

- collection and organization of data (e.g., graphs, tables)
- reading and interpreting statistical information
- decision-making (e.g., for voting or consumer situations)

- measures of central tendency (e.g., mean, median, mode)
- calculating the probability of an event occurring
- ranking procedures
- predicting outcomes

Table 33
Probability and Statistics — Secondary Content

		Content Suitable for			
		None	Non-College Bound	College Bound	All
356.	Probability distributions (e.g., normal, binomial)	6.0	7.5	61.2	19.4
357.	Predicting outcomes	6.0	6.0	35.8	50.7
358.	Curve fitting and prediction	12.1	3.0	54.5	19.7
359.	Measures of central tendency (e.g., mean, median, mode)	6.0	3.0	23.9	61.2
360.	Ranking procedures	9.1	3.0	28.8	51.5
361.	Calculating the probability of an event occurring	0.0	3.0	39.4	54.5
362.	Correlation	25.8	3.0	50.0	15.2
363.	Statistical testing of hypotheses	35.8	4.5	41.8	10.4
364.	Reading and interpreting statistical information	7.5	1.5	11.9	71.6
365.	Decision-making (e.g., for voting or consumer situations)	0.0	1.5	15.2	71.2
366.	Measures of spread (e.g., range, quartiles)	13.4	1.5	34.3	41.8
367.	Combinations and permutations	4.5	0.0	66.7	24.2
368.	Experimental design	32.8	4.5	41.8	9.0
369.	Collection and organization of data (e.g., graphs, tables)	1.5	3.0	10.4	82.1
370.	Calculating probabilities of compound and conditional events	9.0	1.5	68.7	14.9

Goals

All ten goals for probability and statistics stated were accepted as desirable by the respondents. The ranking of the stated goals follows:

- to enable students to read and think critically about graphs and other data in other subjects such as science or social science
- to give experience in dealing with estimation and approximation
- to help consumers deal with statistical information
- to apply mathematics to other disciplines
- to demonstrate how to organize, summarize and present data in easily interpretable forms
- to teach skills necessary for further study
- to teach skills necessary for employment
- to provide practice in such basic mathematical topics as sets, ratio and graphing
- to understand the use and power of computers
- to provide practice in basic computational skills

Table 34
Probability and Statistics — Goals

		HIGH	*LOW*
371.	To help consumers deal with statistical information	85.4	2.2
372.	To enable students to read and think critically about graphs and other data in other subjects such as science or social science	95.5	4.4
373.	To give experience in dealing with estimation and approximation	86.6	2.2
374.	To apply mathematics to other disciplines	84.5	6.7
375.	To understand the use and power of computers	59.6	12.4
376.	To demonstrate how to organize, summarize and present data in easily interpretable forms	80.9	3.3
377.	To provide practice in such basic mathematical topics as sets, ratio and graphing	60.0	21.1
378.	To teach skills necessary for employment	62.2	17.8
379.	To teach skills necessary for further study	76.7	4.4
380.	To provide practice in basic computational skills	58.4	20.2

Resources

The majority of respondents said they would like to have 12 of the 15 curriculum resources listed. These included:

- probability and statistics textbooks that emphasize projects and activities
- resource books with applications and problems from probability and statistics
- individual study materials for students
- a series of short films on videotapes that can be used to motivate and introduce specific probability and statistics concepts
- in-service materials to teach teachers the content of probability and statistics
- masters of worksheets and activities for probability and statistics
- booklets of experiments and related laboratory equipment
- probability and statistics materials for use with small computers
- descriptions of teaching methods appropriate for probability and statistics
- coordinated curriculum materials for probability and statistics encompassing textbooks, laboratory equipment, films, guides, etc.
- a syllabus that suggests probability and statistics topics and methods for each grade level together with specific times when they should be introduced
- a test item bank with test items coordinated to behavioral objectives suitable for probability and statistics

Table 35
Probability and Statistics — Resources

		HIGH	*LOW*
381.	A syllabus that suggests probability and statistics topics and methods for each grade level together with specific times when they should be introduced	74.2	11.3
382.	A series of short films on videotapes that can be used to introduce specific probability and statistics concepts	81.0	7.9
383.	Inservice materials to teach teachers the content of probability and statistics	81.0	6.4
384.	Probability and statistics textbooks that emphasize projects and activities	90.5	3.2
385.	Audiotapes of lectures by eminent statisticians	25.8	50.0
386.	Probability and statistics materials for use with small computers	77.8	11.1
387.	Masters of worksheets and activities for probability and statistics	81.0	7.9
388.	Resource books with applications and problems from probability and statistics	88.3	4.8

389.	Individual study materials for students	85.7	3.2
390.	Descriptions of teaching methods appropriate for probability and statistics	76.2	11.1
391.	Standardized tests in probability and statistics which allow for comparison with students from other schools	53.9	30.1
392.	Coordinated curriculum materials for probability and statistics encompassing textbooks, laboratory equipment, films, guides, etc.	76.2	11.1
393.	A test item bank with test items coordinated to behavioral objectives suitable for probability and statistics	61.9	22.2
394.	Booklets of experiments and related laboratory equipment	79.3	12.7
395.	Outlines of outstanding presentations in probability and statistics	53.9	25.3

Methods

When questioned about strategies for teaching probability and statistics, two-thirds or more of the respondents said the following strategies should be used:

- students perform experiments with dice and cards, and study games of chance
- problems that arise in the social or natural sciences are used to extract and develop probability and statistics concepts
- materials include many examples of real world data such as those taken from newspapers and periodicals
- cases where statistics were misinterpreted or misused are studied
- projects are suggested that are designed to be assigned to individuals or to teams of students
- specific objectives, criterion-referenced testing and other materials are included to encourage use of a mastery learning or an individually paced model
- students are required to analyze data that they have gathered outside the classroom (e.g., street-corner interviews)
- detailed notes are provided to guide the teacher in oral presentations of probability and statistics lessons

Table 36
Probability and Statistics — Methods

		HIGH	*LOW*
396.	Students are required to analyze data that they have gathered outside the classroom (e.g., street-corner interviews)	69.2	19.1
397.	Students perform experiments with dice and cards and study games of chance	85.3	8.8
398.	Materials include many examples of real world data such as those taken from newspapers and periodicals	82.3	7.3
399.	Students are provided with ready-made data bases from previously completed experiments	54.4	26.5
400.	Students are expected to read formal presentations of basic probability and statistics ideas before classroom activities are devoted to these ideas	35.9	46.3
401.	Cases where statistics were misinterpreted or misused are studied	80.9	14.7
402.	Detailed notes are provided to guide the teacher in oral presentations of probability and statistics lessons	66.6	14.7
403.	Problems that arise in the social or natural sciences are used to extract and develop probability and statistics concepts	85.3	1.5
404.	Projects are suggested that are designed to be assigned to individuals or to teams of students	79.5	7.4
405.	Specific objectives, criterion-referenced testing and other materials are included to encourage use of a mastery learning or an individually paced model	70.6	13.2

Who/Time

When questioned about different points in the curriculum when probability and statistics could be introduced, the majority of respondents agreed with the following statements:

- probability and statistics should be offered as part of the general mathematics or consumer mathematics course
- a course in probability and statistics lasting at least one semester should be offered as a high school elective for students who have successfully completed one year of algebra
- probability and statistics should be offered as part of an inter-disciplinary course (e.g., with science or social studies)
- ideas from probability and statistics should be included in every mathematics textbook from grades 1-8
- probability and statistics should be a required course for all ninth graders

Four statements elicited general disagreement:

- statistics belongs in the curriculum but probability does not
- probability belongs in the curriculum but statistics does not
- probability and statistics should only be considered as enrichment topics for mathematics
- probability and statistics should replace most of the traditional work with fractions in grades six, seven and eight

Table 37
Probability and Statistics — Who/Time

		HIGH	*LOW*
406.	Probability and statistics should be a required course for all ninth graders	53.5	33.9
407.	Ideas from probability and statistics should be included in every mathematics textbook from grades one to eight	57.2	33.9
408.	A course in probability and statistics lasting at least one semester should be offered as a high school elective for students who have successfully completed one year of algebra	66.6	15.3
409.	Probability and statistics should only be considered as enrichment topics for mathematics	21.4	67.9
410.	Probability and statistics should be offered as part of the general mathematics or consumer mathematics course	79.2	8.1
411.	Probability and statistics should replace most of the traditional work with fractions in grades six, seven and eight	21.6	67.5
412.	Probability and statistics should be offered as a senior-level advanced course for high ability mathematics and science students	43.2	38.7
413.	Probability and statistics should be offered as part of an inter-disciplinary course (e.g., with science or social studies)	60.3	15.3
414.	Statistics belongs in the curriculum but probability does not	18.9	72.9
415.	Probability belongs in the curriculum but statistics does not	14.4	72.9

Calculators

Five items were used to gather data on how appropriate it was for students to use a calculator while doing various activities in probability and statistics. The majority of respondents agreed that it was appropriate to use the calculator for all five activities. The rank ordering of the activities according to degree of appropriateness follows:

- doing homework in probability and statistics
- calculating the probability that several events will occur in a certain sequence
- taking a probability and statistics test
- calculating the average of the numbers 5, 7, 12, 19 and 23
- making a graph from a number sentence or equation

Table 38
Probability and Statistics — Calculators

		HIGH	*LOW*
416.	Calculating the average of the numbers 5, 7, 12, 19 and 23	51.8	42.4
417.	Making a graph from a number sentence or equation	51.8	32.9
418.	Taking a probability and statistics test	67.9	13.1
419.	Calculating the probability that several events will occur in a certain sequence	70.1	8.4
420.	Doing homework in probability and statistics	73.0	9.4

RATIO, PROPORTION AND PERCENT

Content

The majority of respondents said the following should be included:

- percent should be introduced in a real-life context
- percent should be introduced in terms of merchandising (e.g., discount sales, percent of profit)
- ratio should be introduced as a method for determining the "best buy" in a supermarket or sporting goods store
- a percent, such as sixty-three percent, should be converted to the ratio of 63/100
- ratio and proportion should be developed in connection with similar geometric figures even in non-geometry courses
- direct and indirect variations should be identified as two different patterns when data are graphed
- proportions should be introduced with illustrations of simple chemistry and physics experiments
- each percent problem should be solved by writing an appropriate number sentence
- ratio should be developed as a special kind of fraction before applications of the concept are made
- every percent problem should be solved by setting up a proportion

The item rejected by most was:

- percent should be introduced as a special key on a calculator and the meaning of the concept should be discovered by examining the effects of that key

Table 39
Ratio, Proportion and Percent — Content

		HIGH	*LOW*
421.	Ratio should be introduced as a measure of the "steepness" of different straight-line graphs (e.g., the slope of lines)	46.8	29.9
422.	Percent should be introduced in a real-life context	93.7	2.5
423.	Direct and indirect variations should be identified as two different patterns when data are graphed	57.0	10.1
424.	Percent should be introduced in terms of merchandising (e.g., discount sales, percent of profit)	92.4	5.0
425.	Proportions should be introduced with illustrations of simple chemistry and and physics experiments	57.0	17.7
426.	A percent, such as sixty-three percent, should be converted to the ratio of 63/100	73.1	16.7
427.	Ratio should be introduced as a method for determining the "best buy" in a supermarket or sporting goods store	74.7	16.5
428.	Each percent problem should be solved by writing an appropriate number sentence	56.9	20.2

429.	Every percent problem should be solved by setting up a proportion	53.2	27.9
430.	Ratio should be developed as a special kind of fraction before applications of the concept are made	55.7	26.6
431.	Proportions should be introduced as ways to describe mixtures	40.5	29.2
432.	Proportions should not be dignified by special treatment but as simply a part of equation solving	38.5	39.7
433.	Shortcuts and memory devices should be taught (e.g., "the product of the means equals the product of the extremes")	48.8	33.8
434.	Percent should be introduced as a special key on a calculator and the meaning of the concept should be discovered by examining the effects of that key	20.5	57.7
435.	Ratio and proportion should be developed in connection with similar geometric figures even in non-geometry courses	61.3	22.6

Goals

Worthwhile goals ordered according to preference were:

- to acquire consumer skills such as using percent in analyzing the financing of a purchase such as a new car or a house
- to develop proportional thinking as an important problem solving technique
- to acquire skills necessary for applying mathematics in vocational settings
- to learn special techniques, such as direct and inverse variation, that are powerful tools in sciences such as physics and chemistry
- to demonstrate that ratios provide the foundations for a powerful reasoning process
- to develop, apply and extend the understanding of fractions
- to provide a setting for practicing computational skills

The following goal was not considered to be important by about half of the respondents:

- to preserve a traditional emphasis in the curriculum

Table 40
Ratio, Proportion and Percent — Goals

		HIGH	*LOW*
436.	To preserve a traditional emphasis in the curriculum	30.0	51.1
437.	To provide a setting for practicing computational skills	60.0	25.6
438.	To acquire consumer skills such as using percent in analyzing the financing of a purchase such as a new car or a house	84.4	4.4
439.	To demonstrate that ratios provide the foundations for a powerful reasoning process	68.9	13.3
440.	To learn special techniques, such as direct and inverse variation, that are powerful tools in sciences such as physics and chemistry	74.4	5.6
441.	To develop proportional thinking as an important problem solving technique	80.0	7.8
442.	To identify students who possess mathematical talent	34.8	35.9
443.	To develop, apply and extend the understanding of fractions	62.9	20.2
444.	To acquire skills necessary for applying mathematics in vocational settings	76.9	8.8
445.	To develop and practice disciplined work habits	47.7	22.2

Resources

The majority of respondents agreed that the following resources would be helpful in teaching:

- master copies for making activities and worksheets
- resource books of applications of ratio and percent to real-life problems

- short films and videotapes illustrating basic concepts of ratio and percent
- individualized study materials for ratio and percent
- mathematics laboratory manipulative materials for ratio and percent
- a laboratory book of experiments illustrating ratio and proportion
- charts for reading percents visually
- calculators that allow three numbers of a proportion to be input, with the fourth number calculated and displayed when the "equals" key is pushed

Table 41
Ratio, Proportion and Percent — Resources

		HIGH	*LOW*
446.	Charts for reading percents visually	65.7	19.4
447.	Calculators that allow three numbers of a proportion to be input, with the fourth number calculated and displayed when the "equals" key is pushed	55.9	30.8
448.	Small computers or calculators programmed to handle all three types of percent problems automatically	45.6	33.8
449.	A laboratory book of experiments illustrating ratio and proportion	70.5	11.8
450.	Individualized study materials for ratio and percent	73.2	17.9
451.	Machines for plotting graphs	48.5	25.0
452.	Master copies for making activities and worksheets	85.3	5.9
453.	Mathematics laboratory manipulative materials for ratio and percent	71.6	13.5
454.	Resource books of applications of ratio and percent to real-life problems	72.0	17.7
455.	Short films and videotapes illustrating basic concepts of ratio and percent	77.6	14.9

Methods

The following strategies were endorsed:

- projects involving ratio, proportion and percent are included for assignment to individuals and teams of students
- simple physical experiments to illustrate ratio, proportion and percent are done in a laboratory setting
- student worksheets are included for drill and practice on ratio, proportion and percent topics at the conclusion of each lesson
- detailed notes are provided to guide the teacher in oral presentations of lessons about ratio, proportion and percent
- each ratio, proportion or percent topic is introduced by giving the class a problem
- specific objectives, criterion-referenced testing and other materials are included to encourage the use of mastery learning or an individually paced model
- activities are included that anticipate the class being divided into small discussion groups
- field trips are taken in which students can observe the use of ratio, proportion and percent in business and industry

Table 42
Ratio, Proportion and Percent — Methods

		HIGH	*LOW*
456.	Simple physical experiments to illustrate ratio, proportion and percent are done in a laboratory setting	83.2	3.8
457.	Student worksheets are included for drill and practice on ratio, proportion and percent topics at the conclusion of each lesson	76.7	11.7
458.	Projects involving ratio, proportion and percent are included for assignment to individuals and teams of students	88.4	5.2

459.	Specific objectives, criterion-referenced testing and other materials are included to encourage the use of mastery learning or an individually paced model	65.0	15.6
460.	Field trips are taken in which students can observe the use of ratio, proportion and percent in business and industry	56.6	11.8
461.	Activities are included that anticipate the class being divided into small discussion groups	63.7	18.2
462.	Graphs and charts are used to eliminate as much computation as possible	25.4	44.0
463.	Students are expected to read formal presentations of basic ideas of ratio, proportion and/or percent before classroom activities are devoted to these ideas	39.0	35.1
464.	Each ratio, proportion, or percent topic is introduced by giving the class a problem	68.8	10.4
465.	Detailed notes are provided to guide the teacher in oral presentations of lessons about ratio, proportion and percent	68.9	31.2

Who/Time

With respect to who should study ratio, proportion and percent and at what point in the curriculum, the majority of respondents agreed with the following:

- the mastery of percent problems should be a condition for high school graduation

A majority of respondents disagreed with the following statements:

- only bright students should be taught all three types of percent problems
- most of the work with direct and indirect variation should be handled in science classes rather than mathematics classes
- ratio and proportion should not be introduced until grade nine

Table 43
Ratio, Proportion and Percent — Who/Time

		HIGH	*LOW*
466.	Ratio and proportion should not be introduced until grade nine	31.1	57.8
467.	Only bright students should be taught all three types of percent problems	26.4	67.1
468.	The mastery of percent problems should be a condition for high school graduation	63.4	22.3
469.	More time should be devoted to ratio and proportion than is presently allowed in the school curriculum	44.5	28.9
470.	Most of the work with direct and indirect variation should be handled in science classes rather than mathematics classes	22.3	58.9

Calculators

Most thought it was appropriate to use the calculator in the following ways:

- checking answers
- given that four hours work is needed to product 17 finished brackets, finding how much time is needed to produce 25

Three uses were judged as inappropriate almost as often as appropriate:

- solving the problem, "If 3 cans of corn cost 89¢, what would be the cost of 10 cans of corn?"
- finding the distance from Centerburg to Roseville if the length separating them on a map is 1¼ inches and the map scale is ½ inch = 3 miles
- calculating the final amount owed if an item sells for $15 and the sales tax is 5%

	Table 44 Ratio, Proportion and Percent — Calculators	*HIGH*	*LOW*
471.	Taking a test on ratio, proportion and/or percent	59.5	36.7
472.	Doing homework	55.7	34.2
473.	Calculating the number of dollars saved if a $250 coat is purchased during a 30% discount sale	55.7	35.5
474.	Checking answers	86.1	10.1
475.	Solving the problem, "If 3 cans of corn cost 89¢, what would be the cost of 10 cans of corn?"	49.4	46.9
476.	Finding the distance from Centerburg to Roseville if the length separating them on a map is 1¼ inches and the map scale is ½ inch = 3 miles	46.8	45.6
477.	Finding what percent of $3,000 would yield $50 interest in a period of a year	55.1	32.0
478.	Developing ideas about percents greater than 100%	55.7	26.6
479.	Calculating the final amount owed if an item sells for $15 sales tax is 5%	43.6	50.0
480.	Given that four hours work is needed to produce 17 finished brackets, finding how much time is needed to produce 25	65.9	26.6

PROBLEM SOLVING

Techniques

Respondents felt these five techniques definitely should be included in both elementary and secondary school problem solving:

- construct a table and search for patterns
- write and solve a simpler problem; then extend the solution to the original problem
- teach primarily global problem solving ideas (e.g., read, plan, work, check).
- draw a picture, diagram or graph to represent the problem situation
- translate the problem into number sentences or equations

There was only moderate support for including:

- explore the problem by using flow charts
- start with an approximate answer and work backwards
- guess and test possible solutions

	Table 45 Problem Solving — Elementary Techniques	*HIGH*	*LOW*
481.	Categorize problems into specific types (e.g., age, distance-rate-time), then teach a method of solution for each type	44.2	36.5
482.	Generate many possible answers using a calculator or computer, then check to see which one meets the conditions of the problem	32.7	50.0
483.	Write and solve a simpler problem; then extend the solution to the original problem	82.7	13.4
484.	Explore the problem by using flow charts	69.4	9.6
485.	Translate the problem into number sentences or equations	71.2	15.4
486.	Guess and test possible solutions	53.8	25.0
487.	Start with an approximate answer and work backwards	59.7	23.1

488.	Draw a picture, diagram or graph to represent the problem situation	71.4	8.2
489.	Construct a table and search for patterns	88.5	7.7
490.	Teach primarily global problem solving ideas (e.g., read, plan, work, check)	73.0	9.6

Table 46
Problem Solving — Secondary Techniques

		HIGH	*LOW*
491.	Categorize problems into specific types (e.g., age, distance-rate-time), then teach a method of solution for each type	38.9	42.6
492.	Generate many possible answers using a calculator or computer, then check to see which one meets the conditions of the problem	29.1	47.3
493.	Write and solve a simpler problem; then extend the solution to the original problem	74.1	9.3
494.	Explore the problem either by using flow charts or by writing a computer program	51.9	20.4
495.	Translate the problem into sentences or equations	71.0	18.1
496.	Guess and test possible solutions	55.5	31.5
497.	Start with an approximate answer and work backwards	58.2	23.7
498.	Construct a table and search for patterns	70.3	29.7
499.	Draw a picture, diagram or graph to represent the problem situation	88.7	11.3
500.	Teach primarily global problem solving ideas (e.g, read, plan, work, check)	74.0	13.0

Goals

The major goal selected for problem solving was:

- to develop methods of thinking and logical reasoning

Other goals receiving strong support were:

- to develop the skills to approach new topics in mathematics independently
- to develop creative thought processes
- to enhance the ability to apply mathematics in science
- to acquire skills necessary for living in today's world

Table 47
Problem Solving — Goals

		HIGH	*LOW*
501.	To provide a setting for practicing computational skills	62.1	14.9
502.	To develop methods of thinking and logical reasoning	89.7	6.8
503.	To identify students who possess mathematical talent	58.6	16.0
504.	To learn how to read mathematics	65.8	15.3
505.	To apply recently taught mathematical ideas	70.1	13.7
506.	To acquire skills necessary for living in today's world	75.8	6.8
507.	To develop the skills to approach new topics in mathematics independently	86.0	1.2
508.	To develop creative thought processes	85.1	2.3
509.	To acquire problem solving techniques that are vital to having a well-rounded education	72.1	9.3
510.	To enhance the ability to apply mathematics in science	80.0	3.6

Resources

The favoured resources were:

- a resource guide to real-life problems
- in-service training on problem solving methods for all teachers who teach mathematics
- materials in every class for modeling problems and problem solutions (e.g., graph paper, measuring devices, construction sticks, etc.)
- supplementary materials which contain many more problems like those in textbooks
- card files of problems
- materials for problem solving contests and competitions
- textbook modules for teaching appropriate problem solving strategies

Clearly rejected as a problem solving resource was:

- textbooks with all verbal problems in a single chapter

Table 48
Problem Solving — Resources

		HIGH	*LOW*
511.	Computers for problem exploration	68.0	10.0
512.	Textbook modules for teaching appropriate problem solving strategies (heuristics) at every grade level	80.0	8.0
513.	Inservice training on problem solving methods for all teachers who teach mathematics	91.8	8.1
514.	Supplementary materials which contain many more problems like those in textbooks	88.2	5.9
515.	Resource books with problems that appeal to girls	49.0	25.5
516.	Card files of problems	84.3	5.9
517.	Materials in every class for modeling problems and problem solutions (e.g., graph paper, measuring devices, construction sticks, etc.)	90.2	5.9
518.	Materials for problem solving contests and competitions	82.0	6.0
519.	Laboratory resources outside the school for problem investigation	66.0	10.0
520.	Hand-held calculators for use in problem solving situations	74.5	9.8
521.	Resource books of problems written especially for ethnic minority students	33.4	31.4
522.	Textbooks with all verbal problems in a single chapter	14.0	54.0
523.	More time for mathematics (e.g., longer class periods)	49.0	21.6
524.	Practice tests similar to standardized problem solving tests	66.7	21.5
525.	A resource guide to real-life problems	92.2	5.9

Methods

When asked to indicate degree of support for a variety of methods to use when teaching problem solving, 100% of the respondents agreed that:

- problem assignments are designed to challenge students to think

Also given strong support were:

- projects that involve real-life problem situations should be assigned to individuals or teams of students
- problems are used to introduce mathematical topics
- problems are given in which the use of physical materials will aid in the solution
- students work in small groups to solve problems

Very clearly rejected as methods to use were:

- only problems which students can answer quickly are assigned
- students are expected to read formal presentations about problem solving methods before classroom activities are devoted to these ideas

Table 49
Problem Solving — Methods

		HIGH	*LOW*
526.	Students work in small groups to solve problems	74.4	25.6
527.	Problem assignments are designed to challenge students to think	100.0	0.0
528.	Students are required to create problems and exchange them with one another for solution	64.3	9.4
529.	Reading is de-emphasized by presenting problems orally or with pictures, charts, etc.	28.5	37.7
530.	Problems are included which require more than a single class period to solve	39.6	32.6
531.	More than fifty percent of the instructional time is devoted to drill and practice on problem solving	48.9	37.3
532.	Problems are used to introduce mathematical topics	81.4	7.0
533.	Students are taught to solve problems according to types (e.g., mixture, time-rate-distance)	57.1	26.2
534.	Only problems which students can answer quickly are assigned	2.4	80.9
535.	Projects that involve real-life problem situations should be assigned to individuals or teams of students	95.3	4.6
536.	Students are shown how to solve a problem, then similar practice problems are assigned	73.8	11.9
537.	Students are expected to read formal presentations about problem solving methods before classroom activities are devoted to these ideas	12.2	53.7
538.	Problems are given in which the use of physical materials will aid in the solution	74.4	4.6
539.	Problems are given that do not have exactly one correct answer	57.1	20.4
540.	Specific objectives, criterion-referenced testing and other materials are included to encourage use of a mastery learning or individually paced model	73.1	7.3

Who/Time

In terms of who should receive instruction in problem solving and when, respondents expressed strong agreement that:

- short problem solving units should be included after each mathematical topic is taught
- students should be taught to *find* problems within situations

Strong disagreement was evidenced for these statements:

- problem solving is important only for college-bound students
- problem solving should not be taught in the elementary grades
- problem solving is a function of intelligence and cannot really be taught except to gifted students
- different problem solving courses should be offered for girls

Table 50
Problem Solving — Who/Time

		HIGH	*LOW*
541.	A separate problem solving course, lasting at least one semester, should be required of all students before high school graduation	45.7	44.3
542.	Problem solving is a function of intelligence and cannot really be taught except to gifted students	5.9	92.7
543.	Short problem solving units should be included after each mathematical topic is taught	91.3	8.6
544.	Most students should study practical applications of mathematics; only a few should study puzzles or esoteric mathematical problems	32.3	50.0
545.	Problem solving is important only for college-bound students	0.0	94.2
546.	Different problem solving courses should be offered for girls	4.3	91.4
547.	All problem solving should be done within existing mathematics courses	30.9	61.7
548.	An interdisciplinary problem solving course should be offered	65.2	13.5
549.	Problem solving should not be taught in the elementary grades	1.4	92.7
550.	Students should be taught to *find* problems within situations	86.8	13.2

MEASUREMENT

Content

There was majority agreement that the following should be included in the elementary mathematics measurement strand:

- the metric system
- use of measurement devices (e.g., rulers, protractors, micrometers)
- estimation of measurements
- the use of both non-standard and standard units of measure
- scientific notation (e.g., $5000 = 5 \times 10^3$)
- the multiplication and division of units (e.g., miles/hr x hr = miles)
- significant digits
- formulas for areas of polygons and circles
- history of measurement systems

Of the ten items in this section, only one was rejected by the respondents:

- conversion between different measurement systems

Table 51
Measurement — Elementary Content

		HIGH	*LOW*
551.	Scientific notation (e.g., $5000 = 5 \times 10^3$)	71.9	17.2
552.	The multiplication and division of units (e.g., miles/hr x hr = miles)	69.8	17.4
553.	Conversion between different measurement systems	23.4	64.1
554.	The metric system	100.0	0.0
555.	Significant digits	66.7	12.7
556.	Formulas for areas of polygons and circles	59.4	23.5
557.	Use of measurement devices (e.g., rulers, protractors, micrometers)	93.8	6.3
558.	The use of both non-standard and standard units of measure	78.1	11.0
559.	Estimation of measurements	89.0	6.3
560.	History of measurement systems	57.8	21.9

With respect to topics which should or should not be included in the secondary school measurement strand, the majority of respondents said the following topics should be included:

- the metric system
- estimation and approximation of measures
- the multiplication and division of units (e.g., miles/hr x hr = miles)
- angle and arc measurement
- significant digits
- formulas for areas of polygons and circles
- the use of arbitrary and standardized units of measure
- formulas for distance on the coordinate plane

Table 52
Measurement — Secondary Content

		HIGH	*LOW*
561.	Angle and arc measurement	74.6	19.1
562.	The multiplication and division of units (e.g., miles/hr x hr = miles)	77.4	14.5
563.	Conversion between similar units in different systems	42.8	49.2
564.	The metric system	100.0	0.0
565.	Significant digits	69.8	19.1
566.	Formulas for areas of polygons and circles	69.8	25.4
567.	Formulas for distance on the coordinate plane	54.0	38.1
568.	The use of arbitrary and standardized units of measure	61.3	19.3
569.	Estimation and approximation of measures	98.4	1.6
570.	History of measurement systems	32.3	46.7

Goals

The following goals for measurement were accepted as valid:

- to develop and practice estimation skills
- for everyday use in the home (e.g., comparisons, decisions)
- to develop skills that are prerequisite to other school work such as science or mathematics
- to acquire skills necessary for living in today's world
- to learn to use specific tools for measurement (e.g., protractors, rulers, micrometers, calipers, scales)
- to give meanings to the numbers that are used in arithmetic
- to develop job-oriented skills
- to provide laboratory experiences

Table 53
Measurement — Goals

		HIGH	*LOW*
571.	To develop skills that are prerequisite to other school work such as science or mathematics	86.8	2.7
572.	To give meanings to the numbers that are used in arithmetic	73.0	6.0
573.	To provide laboratory experiences	65.2	8.7
574.	To develop physical coordination	35.9	26.3
575.	To learn to use specific tools for measurement (e.g., protractors, rulers, micrometers, calipers, scales)	80.0	9.5
576.	To relate mathematics to historical and cultural developments	47.9	20.0
577.	For everyday use in the home (e.g., comparisons, decisions)	87.7	2.
578.	To acquire skills necessary for living in today's world	86.8	3.
579.	To develop job-oriented skills	66.7	11.4
580.	To develop and practice estimation skills	88.7	3.5

Resources

Of the ten resources listed for teaching measurement, respondents said it would be good to have the following eight:

- resource books with problems involving the application of measurement concepts
- student booklets of experiments or activities
- a basic kit of measuring tools for each student
- individual study materials for measurement
- masters of worksheets and activities
- large-scale measuring devices for teacher demonstrations
- films or videotapes showing basic measuring processes
- videotaped interviews with craftsmen and workers describing how they use measurement on the job

Table 54
Measurement — Resources

		HIGH	*LOW*
581.	A basic kit of measuring tools for each student	81.1	10.1
582.	Films or videotapes showing basic measuring processes	75.0	23.2
583.	Masters of worksheets and activities	78.3	10.1
584.	Electronic measuring tools that show all measurements on a digital display similar to that of a calculator	53.6	18.6
585.	Individual study materials for measurement	80.9	4.4
586.	Calculators with special keys for converting between measurement systems	30.4	40.6
587.	Large-scale measuring devices for teacher demonstrations	78.3	14.4
588.	Student booklets of experiments or activities	84.1	4.3
589.	Videotaped interviews with craftsmen and workers describing how they use measurement on the job	73.9	13.0
590.	Resource books with problems involving the application of measurement concepts	88.4	4.3

Methods

Of the ten strategies for teaching measurement, the respondents supported the following:

- assignments for students or teams of students include projects that require measurement
- student worksheets are included for drill and practice on measurement topics at the conclusion of each lesson
- each measurement topic is introduced by giving the class a problem
- basic measurement ideas are introduced through laboratory investigations
- simulations, wherein each student plays the role of a consumer or worker using measurement in real world situations, are frequently included
- activities are included that would require students to go outside the classroom to measure things
- specific objectives, criterion-referenced testing and other materials are included to encourage use of a mastery learning or an individually paced model
- detailed notes are provided to guide the teacher in oral presentations of lessons about measurement
- presentations and discussions of measurement techniques are given before students actively measure

Table 55
Measurement — Methods

		HIGH	*LOW*
591.	Activities are included that would require students to go outside the classroom to measure things	67.2	17.2
592.	Assignments for students or teams of students include projects that require measurement	79.7	6.2
593.	Presentations and discussions of measurement techniques are given before students actively measure	60.9	15.6
594.	Basic measurement ideas are introduced through laboratory investigations	68.8	6.3
595.	More than fifty percent of the instructional time is devoted to individual study materials to develop and extend measurement ideas	40.3	21.0
596.	Simulations, wherein each student plays the role of a consumer or worker using measurement in real world situations, are frequently included	68.8	6.3
597.	Student worksheets are included for drill and practice on measurement topics at the conclusion of each lesson	77.8	7.9
598.	Each measurement topic is introduced by giving the class a problem	76.6	7.8
599.	Detailed notes are provided to guide the teacher in oral presentations of lessons about measurement	61.3	12.9
600.	Specific objectives, criterion-referenced testing and other materials are included to encourage use of a mastery learning or an individually paced model	65.6	10.9

Who/Time

Respondents agreed with the following statements about where and when measurement should be taught:

- work on measurement should appear at every level from K-8
- measurement should be a strong focus of ninth grade general mathematics or consumer mathematics

Respondents disagreed with the following statements:

- topics in measurement should not be introduced before junior high school
- all work in measurement should be taught by science teachers or in the context of science lessons

Table 56
Measurement — Who/Time

		HIGH	*LOW*
601.	All work in measurement should be taught by science teachers or in the context of science lessons	20.7	74.5
602.	Work on measurement should appear at every level from K-8	89.6	7.6
603.	Topics in measurement should not be introduced before junior high school	7.5	88.7
604.	Measurement should be a major theme of geometry	44.2	35.6
605.	Measurement should be a strong focus of ninth grade general mathematics or consumer mathematics	55.2	24.8

Calculators

When dealing with problems and calculations in measurement, the majority of respondents said it was appropriate to use the calculator in the following settings:

- checking answers
- calculating the diameter of a tree after measuring its circumference
- finding the total length of a road rally course given the odometer readings at various checkpoints
- converting from one system of units to another
- finding the number of rolls of wallpaper necessary to cover the walls of a room whose dimensions are given
- finding the number of gallons of water a swimming pool will hold
- doing homework problems involving measurements
- finding the area of the opening of a fireplace 125 cm tall and 205 cm wide

About sixty-one percent said that it was inappropriate to use the calculator when:

- finding the volume of a rectangular shipping crate, 2 ft. x 4 ft. x 5 ft.

Table 57
Measurement — Calculators

		HIGH	*LOW*
606.	Finding the number of gallons of water a swimming pool will hold	66.7	17.9
607.	Taking a test on measurement	49.4	37.7
608.	Converting from one system of units to another	69.6	20.2
609.	Calculating the diameter of a tree after measuring its circumference	76.0	13.9
610.	Doing homework problems involving measurements	59.4	25.3
611.	Finding the volume of a rectangular shipping crate 2 ft. x 4 ft. x 5 ft.	32.0	61.5
612.	Finding the number of rolls of wallpaper necessary to cover the walls of a room whose dimensions are given	68.0	24.4
613.	Checking answers	84.6	10.3
614.	Finding the area of the opening of a fireplace 125 cm tall and 205 cm wide	53.1	32.9
615.	Finding the total length of a road rally course given the odometer readings at various checkpoints	70.9	17.7

COMPUTER LITERACY

Content

Strong support was indicated for the inclusion of the following six topics in the curriculum:

- the roles of computers in society (record-keeping, simulation, etc.)
- the types of mathematical and non-mathematical problems that can be solved by a computer
- procedures for accessing or operating a computer system
- flow charting
- operation of a programmable calculator
- writing programs in a simple computer language such as BASIC

Table 58
Computer Literacy — Content

		HIGH	*LOW*
616.	Procedures for accessing or operating a computer system	80.4	9.8
617.	Memory storage and access systems	66.0	14.0
618.	Writing programs in a simple computer language such as BASIC	76.7	23.3
619.	The roles of computers in society (record-keeping, simulation, etc.)	89.3	5.8
620.	Methods for debugging or correcting computer programs	48.6	51.4
621.	Issues of privacy and security raised by computers	67.0	12.6
622.	The functioning of microprocessor units	41.2	19.6
623.	Flow charting	79.6	7.8
624.	The use of machine language	59.8	20.5
625.	History of computing devices	51.5	16.5
626.	Languages for non-computational programs (e.g., Course Writer, PLATO)	26.7	29.7
627.	Operation of a programmable calculator	78.7	21.4
628.	The types of mathematical and non-mathematical problems that can be solved by a computer	84.0	7.0
629.	Computational programming langage (e.g., FORTRAN, COBOL)	46.0	18.6
630.	Data processing for business application (e.g., billing, inventory control)	63.1	12.6

Goals

All five goal statements were endorsed in rank order as follows:

- to develop logical thinking abilities
- to prepare for the Twenty-First century in which almost everyone will be interacting directly with a computer programmable device
- to acquire fundamental computer techniques necessary for vocational training
- to understand the capability of the computer to provide access to large bodies of information (e.g., information retrieval systems)
- to introduce alternative techniques for solving problems, proving theorems, etc.

Table 59
Computer Literacy — Goals

		HIGH	*LOW*
631.	To acquire fundamental computer techniques necessary for vocational training	72.6	14.7
632.	To prepare for the Twenty-First century in which almost everyone will be interacting directly with a computer programmable device	77.9	7.4
633.	To introduce alternative techniques for solving problems, proving theorems, etc.	64.2	10.5
634.	To understand the capability of the computer to provide access to large bodies of information (e.g., information retrieval systems)	70.3	8.5
635.	To develop logical thinking abilities	78.9	6.4

Resources

Four of the five resources listed were deemed desirable by at least seventy percent of those responding:

- several small, personal mini-computers for each class
- wall-sized demonstration screens connected to computers for video output
- workbooks with paper and pencil algorithms simulating computer processes
- a terminal connected to a large computer

Table 60
Computer Literacy — Resources

		HIGH	*LOW*
636.	Wall-sized demonstration screens connected to computers for video output	73.5	10.2
637.	A terminal connected to a large computer	70.0	10.0
638.	Several small, personal mini-computers for each class	75.0	3.8
639.	Equipment for batch processing (e.g., card readers, paper tape punches, magnetic tapes and disks)	58.0	16.0
640.	Workbooks with paper and pencil algorithms simulating computer processes	72.5	13.8

Methods

Over seventy percent of the respondents rated four methods as acceptable:

- students are assigned to individual projects to study different computer applications and the impact of these applications
- programming techniques are taught by computer-assisted instruction in tutorial mode
- field trips are taken in which students can observe the use of computers in business and industry
- more than fifty percent of the instructional time is devoted to student use of individual study materials to develop and extend computer ideas

Table 61
Computer Literacy — Methods

		HIGH	*LOW*
641.	Cases where the computer was misused are studied	37.9	24.1
642.	Students are expected to read formal presentations of computer ideas before classroom activities are devoted to these ideas	32.1	39.8
643.	Program writing is taught by a trial and error approach that emphasizes discovery of fundamental programming principles	46.3	27.5
644.	Programming techniques are taught by computer-assisted instruction in tutorial mode	77.5	3.8
645.	Students are assigned to individual projects to study different computer applications and the impact of these applications	87.6	12.5
646.	Computer ideas are taught by simulations using large scale devices to demonstrate how a computer works	68.8	12.6
647.	Field trips are taken in which students can observe the use of computers in business and industry	73.8	10.1
648.	Detailed notes are provided to guide the teacher in oral presentations of computer topics	68.8	11.3
649.	At least fifty percent of the instructional time is devoted to students writing computer programs	68.3	8.8
650.	More than fifty percent of the instructional time is devoted to student use of individual study materials to develop and extend computer ideas	70.9	12.7

Who/Time

Ten items addressed themselves to the sequencing and student audience for computer applications in the school mathematics curriculum. Four items drew agreement from more than fifty percent of those asked:

- at least one course whose major theme is computer literacy and which lasts for at least one semester should be required of all high school graduates

- computer literacy topics should be integrated within the present mathematics curriculum from grades K-12
- because the computer techniques needed for vocational training are different from those needed by college-bound students, at least two different types of computer courses should be offered in every high school
- all high school graduates should be able to write simple computer programs

Strong disagreement by more than sixty percent of those asked was registered on two items.

- computer literacy courses should be taught primarily within the social studies curriculum since it is the effect of computers upon society that is important
- knowledge of computers is only needed by specialists; they should receive courses and training in this area only after they leave high school

Table 62
Computer Literacy — Who/Time

		HIGH	*LOW*
651.	Computer literacy courses should be taught primarily within the social studies curriculum since it is the effect of computers upon society that is important	15.5	66.0
652.	At least one course whose major theme is computer literacy and which lasts for at least one semester should be required of all high school graduates	70.6	21.6
653.	Students should interact with a computer or computer terminal as early as the primary grades	35.9	41.8
654.	Separate computer science departments should be established in high schools to parallel mathematics departments and science departments	36.0	37.9
655.	All high school graduates should be able to write simple computer programs	50.5	33.6
656.	Computer literacy topics should be integrated within the present mathematics curriculum from grades K-12	63.1	25.0
657.	Because the computer techniques needed for vocational training are different from those needed by college-bound students, at least two different types of computer courses should be offered in every high school	59.4	26.4
658.	Knowledge of computers is only needed by specialists; they should receive courses and training in this area only after they leave high school	39.3	61.9
659.	Courses about computers should be strictly elective	48.9	35.5
660.	Computer courses should use a wide variety of hardware with instructions in the use of each type forming a major part of the course	44.0	30.8

LAY OPINION

The preferences forms for the lay group contained only items relating to the six specific topics, across all the content strands. The lay sample included secondary principals, school board members, parents and members of the business community.

Content

One-half the lay population received a set of statements regarding the content of school mathematics and respondents were asked to indicate their degree of agreement with each statement.

Strong agreement was expressed for the following statements by over 75 percent of the respondents:

- work on measurement should be taught in the elementary school
- a student should know whole-number computation with paper and pencil before graduating from high school
- mental calculations, without the aid of paper and pencil or calculator should be taught
- knowing how to do percent problems should be a condition for high school graduation
- learning about what computers can and cannot do should be integrated within the present mathematics curriculum from grades K-12

Moderate agreement was indicated for:

- a full-year course in applied geometry (for example, navigation, measurement) should be available in high school
- algebra should be combined with geometry and other mathematical areas instead of being taught separately
- at least one course whose major theme is the role and uses of computers, lasting at least one semester, should be required for high school graduation
- college-bound high school students should spend at least three weeks of every school year reviewing whole-number computation

Strong disagreement was evidenced for:

- formal work with algebra should be dropped from the school curriculum since it bears so little relation to real world problems
- mathematical problem solving is more important for college-bound students than for other students

Table 63
Content — Lay Opinion I

		HIGH	*LOW*
743.	Operations with decimals should be included in the first or second-grade mathematics program (the earlier the better)	45.3	47.6
744.	Algebra should be combined with geometry and other mathematical areas instead of being taught separately	57.4	25.7
745.	A student should know whole-number computation with paper and pencil before graduating from high school	91.6	1.2
746.	A full-year course in applied geometry (for example, navigation, measurement) should be available in high school	63.1	22.7
747.	Knowing how to do percent problems should be a condition for high school graduation	85.7	13.1
748.	Mathematical problem solving is more important for college-bound students than for other students	20.4	71.1
749.	Work on measurement should be taught in the elementary school	92.9	2.4
750.	Work with decimals is more important than work with fractions	37.3	40.9

751.	Formal work with algebra should be dropped from the school curriculum since it bears so little relation to real world problems	14.1	74.1
752.	Mental calculations, without the aid of paper and pencil or calculator should be taught	85.9	9.4
753.	Dealing with statistical information should be taught only as an enrichment topic for mathematics	45.8	40.0
754.	Learning about what computers can and cannot do should be integrated within the present mathematics curriculum from grades K-12	75.3	17.6
755.	College-bound high school students should spend at least three weeks of every school year reviewing whole-number computation	48.8	23.8
756.	More of the mathematics curriculum in grades seven and eight should be devoted to geometry	34.1	33.0
757.	At least one course whose major theme is the role and uses of computers, lasting at least one semester, should be required for high school graduation	57.6	28.3

The other half of the lay group received a different, although related, set of statements about the content of school mathematics. Again, respondents were asked to indicate their degree of agreement with each statement.

The most strongly supported statement was:

- specific consumer skills like balancing a checkbook and calculating "best buys" should be taught

Receiving support from over 70 percent of the total group were:

- percent should be introduced in terms of merchandising (e.g., discount sales, percent of profit)
- problems should be realistic even though they might involve sensitive social issues
- work with statistical information and making predictions (probability) should be offered as a twelfth-grade course for high ability mathematics students
- at least one course in mathematics for college-bound students should make extensive use of the computer

Some disagreement was evident in regard to:

- all fractions should be written as decimals so that work with them can be done with a calculator

Strong disagreement was indicated for:

- work with fractions should be delayed until grade seven or eight

Table 64
Content — Lay Opinion II

		HIGH	*LOW*
769.	All fractions should be written as decimals so that work with them can be done with a calculator	28.2	61.6
770.	A special algebra course for vocational students should be offered	62.8	16.3
771.	Several different procedures for doing addition, subtraction, multiplication and division should be taught so that children can choose the method which they prefer	49.7	43.3
772.	Separate courses in geometry should be abolished and geometry content integrated with other mathematics in grades K-12	52.9	26.1

773.	Percent should be introduced in terms of merchandising (e.g., discount sales, percent of profit)	74.1	9.3
774.	A separate problem solving course, lasting at least one semester should be required of all students before high school graduation	59.7	29.8
775.	Measurement should be a strong focus of consumer mathematics courses in high school	68.0	14.2
776.	Work with fractions should be delayed until grade seven or eight	11.8	78.7
777.	A student graduating from high school should be required to take a full-year algebra course	45.7	35.9
778.	Students who do not know paper and pencil computation by the end of grade eight should be required to take a ninth grade mathematics course on the uses of the hand-held calculator	42.3	44.0
779.	Work with statistical information and making predictions (probability) should be offered as part of a consumer mathematics course	61.2	17.7
780.	At least one course in mathematics for college-bound students should make extensive use of the computer	71.0	15.9
781.	Specific consumer skills like balancing a checkbook and calculating "best buys" should be taught	94.2	2.3
782.	Problems should be realistic even though they might involve sensitive social issues	72.8	8.3
783.	Work with statistical information and making predictions (probability) should be offered as a twelfth-grade course for high ability mathematics students	72.7	13.4

Goals

When asked to consider the reasons for teaching mathematics in schools, over eighty percent of the lay group indicated these goals were important:

- to think logically
- to solve problems in everyday life
- to gain skills necessary for employment
- to make consumer decisions
- to preserve student options with respect to potential careers and vocational choices
- to develop disciplined work habits
- to teach skills necessary for continued work in mathematics
- to develop understanding of the structure of mathematics

Moderate importance was attached to:

- to develop for college
- to assure an adequate supply of scientists and engineers

Clearly rejected as a goal was:

- to preserve a traditional part of schooling

Respondents were divided in their opinions on only one item, with about forty-eight percent indicating it was an important goal and thirty-four percent indicating it was an unimportant goal:

- to pass standardized tests

Table 65
Goals — Lay Opinion

		HIGH	*LOW*
729.	To solve problems in everyday life	94.6	3.1
730.	To think logically	96.9	0.4
731.	To assure an adequate supply of scientists and engineers	66.8	16.6
732.	To preserve student options with respect to potential careers and vocational choices	86.2	4.6
733.	To develop understanding of the structure of mathematics	80.9	7.6
734.	To develop disciplined work habits	83.6	7.7
735.	To develop for college	75.8	10.0
736.	To pass standardized tests	48.2	34.0
737.	To gain skills necessary for employment	92.0	2.7
738.	To make consumer decisions	89.0	4.9
739.	To preserve a traditional part of schooling	27.4	60.5
740.	To teach skills necessary for continued work in mathematics	82.9	8.4

Resources

Six statements about resources for use in classrooms received very strong support:

- short problem solving sections are included after each mathematical topic is taught
- tests, homework and specific objectives are included to encourage each student to attain a specified competency level
- ideas or procedures are developed through real-life problems, situations, or activities
- physical materials, which the students can manipulate to help them understand mathematical ideas, are included in many lessons
- students' worksheets or workbooks are included for drill and practice at the conclusion of each lesson
- activities are included that anticipate the class being divided into small groups

Moderate support was given to:

- the introduction of calculators is postponed until students have learned both the meaning of and paper and pencil procedures for, whole-number computation
- detailed notes are provided to guide the teacher in oral presentations of lessons

Strong disagreement was evident in regard to:

- only problems which students can answer quickly are assigned
- reading is de-emphasized in textbooks and other materials

Table 66
Resources — Lay Opinion

		HIGH	*LOW*
758.	Students' worksheets or workbooks are included for drill and practice at the conclusion of each lesson	82.9	8.6
759.	Activities are included that anticipate the class being divided into small groups	79.6	9.6
760.	Tests, homework and specific objectives are included to encourage each student to attain a specified competency level	96.4	0.0
761.	Only problems which students can answer quickly are assigned	7.3	90.1

762.	The introduction of calculators is postponed until students have learned both the meaning of, and paper and pencil procedures for, whole-number computation	79.2	13.3
763.	Reading is de-emphasized in textbooks and other materials	16.7	70.5
764.	Ideas or procedures are developed through real-life problems, situations, or activities	92.6	1.3
765.	Detailed notes are provided to guide the teacher in oral presentations of lessons	70.6	15.3
766.	Short problem solving sections are included after each mathematical topic is taught	96.4	0.0
767.	Physical materials, which the students can manipulate to help them understand mathematical ideas, are included in many lessons	89.3	3.6

Methods

Two teaching strategies were very strongly supported:

- students are shown how to solve a problem and then similar practice problems are assigned
- activities are included which require going outside the classroom

Also receiving strong support were:

- basic ideas are introduced through laboratory investigations or experiments with materials
- each new mathematical topic is introduced with a problem to be solved
- more than fifty percent of the instructional time is devoted to drill and practice
- daily homework assignments are included

Two strategies received mixed responses, with about thirty percent disagreeing with the strategy but about fifty percent in favor:

- slower students are allowed to use calculators so that they may keep up with the rest of the class
- most lessons are designed to be conducted with a single large group

Table 67
Methods — Lay Opinion

		HIGH	*LOW*
784.	More than fifty percent of the instructional time is devoted to drill and practice	69.4	18.5
785.	Slower students are allowed to use calculators so that they may keep up with the rest of the class	47.5	33.1
786.	Activities are included which require going outside the classroom	79.6	6.7
787.	Each new mathematical topic is introduced with a problem to be solved	72.8	10.8
788.	More than fifty percent of the instructional time is devoted to student use of individual study materials to develop and extend ideas	54.8	21.8
789.	Basic ideas are introduced through laboratory investigations or experiments with materials	73.0	8.6
790.	Students are to read about mathematical ideas before classroom activities are devoted to these ideas	45.0	23.1
791.	Most lessons are designed to be conducted with a single large group	50.6	28.8
792.	Daily homework assignments are included	69.2	14.2
793.	Students are shown how to solve a problem and then similar practice problems are assigned	82.4	7.7

Who/Time

Respondents were asked how many years of high school (grades nine through twelve) mathematics they would require for graduation, with the choices being zero through four years. About fifty percent of the total group indicated four years and thirty percent of the group indicated three years. Asked the same question about college-bound students, the percentages increased with eighty percent indicating four years and about sixteen percent indicating three years. Those from the business community consistently opted for more mathematics.

Table 68
Years of High School Mathematics Required for Graduation

YEARS	*PERCENT RESPONSE*
0	3.5
1	2.4
2	11.0
3	31.9
4	51.2

Table 69
Years of High School Mathematics Required for College-Bound Students

YEARS	*PERCENT RESPONSE*
0	1.2
1	0.8
2	1.9
3	15.9
4	80.2

Calculators

Six activities were presented and respondents asked to indicate how appropriate it is for students to use calculators when doing each activity.

Only one of the six was considered appropriate:

- checking answers

Roughly thirty-three percent of the respondents considered the following activities appropriate for calculator use, but forty to fifty percent considered them inappropriate:

- taking a test
- solving word problems

The opposite was true for another two items; fifty percent considered them appropriate and thirty-three percent inappropriate:

- developing ideas and concepts
- doing practice exercises in class

Respondents were about evenly divided on the appropriateness of using the calculator when:

- doing homework

Table 70
Calculators — Lay Opinion

		HIGH	*LOW*
661.	Doing homework	47.8	41.2
662.	Developing ideas and concepts	50.4	33.8
663.	Taking a test	30.2	51.1
664.	Checking answers	86.8	8.1
665.	Solving word problems	33.5	41.9
666.	Doing practice exercises in class	50.6	36.8

III. PRIORITIES SURVEY

The purpose of the Priorities Survey was to assess professional and lay priorities with respect to a number of issues in school mathematics. There were two typical item forms. One type asked the respondent to rank order five alternatives usually in response to questions of expenditure of money, allowance of extra time or material development. This was followed by a second item asking the reason for the ranking. The findings reported are based upon an analysis of the choices made by professionals and lay persons when confronted with these two types of items. It might be said that the priority items were general and conservative in nature. They did, however, allow a wide variety of respondents to react without need for understanding special technical language or terms.

There were not a large number of high or low priorities evidenced with respect to curriculum and course issues. Items highlighted below reflect priorities, where a significant proportion of a sample rated the items high or low. In the tables following the highlighted items, percentages indicate only the highest and lowest priorities from a five-point scale, for each of the professional and lay groups and for the total sample. The fact that there are relatively few strong priorities may result from a satisfaction with the current mathematics curriculum. Indeed, eighty-five percent of the professional and lay groups thought the mathematics curriculum was as good or better than other school curriculums.

ELEMENTARY CURRICULUM

Spending

When asked to attach priorities to extra spending on five elementary curriculum areas, both the professional and lay groups rated problem solving as a high priority area for further expenditures. The reason both groups gave was the crucial nature of the skill. Because they thought whole numbers were crucial, the lay groups gave whole numbers a high priority. A strong low priority ranking was given common fractions by the professionals who saw this topic as being of diminished importance.

Table 71

Elementary Curriculum — Spending

Imagine that you have a limited amount of money to spend for development of new materials for elementary grades, in the areas listed below. Please indicate the order in which you think the money should be spent.

Item		*Professional*	*Lay*	*Total*
Problem Solving	High	59.9	46.6	56.8
	Low	3.1	1.1	2.6
Measurement	High	5.5	9.3	6.3
	Low	12.3	33.7	17.2
Fractions	High	1.7	4.7	2.4
	Low	48.6	23.3	42.9
Decimals	High	13.2	9.3	12.3
	Low	8.8	23.3	12.1
Whole Number Computation	High	20.9	39.5	25.1
	Low	23.0	9.3	19.9

Time

When asked to identify a curriculum priority for an extra fifteen minutes of study, both lay and professional groups opted for applications as high priority. While seventy percent of the lay population felt drill and practice should be a first or second priority for such extra time, the professionals were divided with twenty-seven percent rating this as highest and another twenty-three percent rating it lowest. Both groups felt enrichment topics and an intuitive base for algebra and geometry would be a low priority for this extra fifteen minutes.

Table 72
Elementary Curriculum — Time

Suppose that an additional fifteen minutes each day could be spent on mathematics in your elementary school(s). In your opinion, how should this time be spent?

Item		*Professional*	*Lay*	*Total*
Word Problems	High	31.3	17.0	28.0
	Low	5.2	14.8	7.4
Drill and Practice	High	27.3	46.1	31.7
	Low	23.5	6.7	19.6
Enrichment	High	10.9	4.7	9.5
	Low	25.3	29.1	26.1
Application	High	22.1	27.0	23.2
	Low	3.8	5.6	4.2
Algebra and Geometry Intuition	High	11.3	5.9	10.1
	Low	37.9	36.5	37.6

SECONDARY CURRICULUM

Spending

The rank ordering of expenditures for secondary curriculum topics saw both professional and lay groups give highest priority to computer literacy. Both groups saw it as becoming increasingly important in the 80's, while the professionals also noted the lack of suitable instructional materials.

There was an interesting priority split on two items. Lay persons rated algebra a high priority because it was crucial while professionals rated it a low priority for new action, as the curriculum was seen as adequate. In an opposite vein, the professionals saw statistics as increasingly important and gave it a limited priority support, while statistics was ranked low or unimportant by lay persons.

Table 73
Secondary Curriculum — Spending

Imagine that you have a limited amount of money to spend for the development of new materials for grades seven to twelve in the areas listed below. Please indicate the order in which you think the money should be spent.

Item		*Professional*	*Lay*	*Total*
Computer Literacy	High	40.2	37.6	39.6
	Low	15.3	14.1	15.0
Algebra	High	25.4	39.5	28.6
	Low	34.9	8.1	28.6

Probability	High	3.9	14.1	6.3
	Low	22.7	23.5	22.9
Geometry	High	17.9	4.7	14.9
	Low	12.1	12.8	12.2
Statistics	High	13.8	4.8	11.7
	Low	13.1	32.1	17.5

New courses

The support for computer literacy did not manifest itself in terms of priority for a new course for secondary schools. There was support for courses in consumer and career mathematics as well as applications in both groups, which certainly reflects the current status of these concerns both in and out of schools. Calculus as a course received lowest priority ranking reflecting little interest in this and also a feeling among the professionals that this was handled well already in the curriculum. Advanced placement (university level) courses ranked low while general enrichment ranked high.

Table 74
Secondary Curriculum — New Courses

Suppose that *one* new or *extensively* revised mathematics course could be added to the curriculum of your high school(s). In what order would you support the following choices?

Item		*Professional*	*Lay*	*Total*
Ideas from calculus	High	12.3	13.6	12.6
	Low	54.7	39.8	51.2
Use of statistics data and prediction	High	11.5	5.4	9.2
	Low	16.4	21.6	18.4
Consumer mathematics	High	25.9	41.1	29.5
	Low	9.4	6.7	8.8
How calculators and home computers handle mathematics	High	21.7	2.7	14.4
	Low	18.3	24.3	20.6
Vocational and career mathematics	High	15.1	34.8	19.8
	Low	15.8	6.7	13.7

Gifted students

With respect to more mathematics for gifted high school students, offering a broad selection of enrichment topics was given very strong priority over offering additional topics in algebra, geometry, calculus, or numerical analysis.

Table 75
Secondary Curriculum — Gifted Students

If more mathematics were offered to accomodate talented or gifted students at the high school level, which *one* of the following would be most important?

Item	*Professional*	*Lay*	*Total*
Additional topics in geometry	3.8	6.7	4.5
Topics in calculus and analysis	5.8	14.6	7.9
Additional topics in algebra	2.4	3.4	2.6
Computers/numerical analysis	17.8	15.7	17.3
A broad selection of enrichment topics	70.2	59.6	67.7

Special courses

When asked to consider special courses or approaches, the total group gave highest priority to applications approaches. The professional group ranked unified and interdisciplinary approaches high while the lay group ranked these low, an apparent contradiction to the applications priority. The professional group ranked structural approaches low, thinking that this was well done already.

Table 76
Secondary Curriculum — Special Courses

In developing curriculum, should attention be given to developing special courses which emphasize:

Item		*Professional*	*Lay*	*Total*
Unified approaches to mathematics topics	High	32.3	24.3	29.3
	Low	9.7	27.0	16.2
Roles and uses of computer	High	26.8	18.9	24.9
	Low	20.9	22.2	21.2
Applications	High	35.0	40.4	36.3
	Low	2.1	1.1	1.9
Structure in mathematics	High	8.0	13.8	9.4
	Low	44.8	31.0	41.6
Interdisciplinary mathematics and science	High	8.5	8.0	8.4
	Low	14.9	23.9	17.1

Orientations

Finally, the groups were asked to rank five orientations to mathematics. Vocational orientations received very high priority from the lay group and high priority from the professional group, as did consumer orientation from both groups. Both groups gave very low priority to recreational and leisure mathematics orientations.

Table 77
Secondary Curriculum — Orientations

Of the five areas listed below, how should attention be given during the 1980's?

Item		*Professional*	*Lay*	*Total*
Career or vocation orientation	High	32.3	65.2	40.2
	Low	7.8	1.1	6.2
Consumer orientation	High	30.9	19.1	28.1
	Low	5.9	4.5	5.6
College preparatory orientation	High	11.1	6.7	10.1
	Low	26.3	6.7	21.7
Recreation or leisure-time orientation	High	4.2	0.0	3.2
	Low	43.3	60.7	47.4
Computer orientation	High	21.8	10.0	19.0
	Low	17.0	22.2	18.2

STUDENT NEEDS

When considering which student needs required most attention, the highest priority for both groups was for curriculum for the handicapped, which both lay and professional saw as an addressable need, but with few good materials available. Surprisingly, given the current research concerns in the area, both groups rated special curriculum for women a low priority.

Table 78
Student Needs

People have various opinions about the extent to which needs of various types of students are being met. Please indicate the order in which you think the need should be addressed.

Item		*Professional*	*Lay*	*Total*
Students whose first language is not English (or French)	High	5.1	7.0	5.6
	Low	16.1	10.5	14.7
Inner-city or urban-area students	High	12.8	12.8	12.8
	Low	6.6	10.5	7.5
Students of ethnic minority background	High	2.9	5.7	3.6
	Low	10.2	11.8	10.5
Students with mathematics learning problems and other handicaps	High	77.6	68.5	75.4
	Low	2.9	2.2	2.7
Female students	High	2.9	3.5	3.1
	Low	67.4	67.4	67.4

CONTENT IMPROVEMENT

The low priority for courses for special groups was reinforced in response to another question where sixty percent saw a need for emphasis on curriculum for the general population, twenty-eight percent supported emphasis on vocational groups and twelve percent looked for curriculum emphasis for university students.

Table 79
Content Improvement

In which of the following areas do you feel your mathematics program needs the *most* improvement?

	Professional	*Lay*	*Total*
Mathematics for general education	60.6	59.1	60.2
Mathematics for the college-bound student	11.6	12.5	11.8
Mathematics for the vocational student	27.5	28.4	27.7

TEACHER EDUCATION

Respondents were asked what priorities should be given to various aspects of teacher education during the 1980's. Methods for teaching mathematics was the first priority of the total group, followed closely by sensitivity to student needs. With the second highest priority choices taken into account, each received nearly sixty percent of the high priority

responses. Diagnostic and remediation strategies was ranked third and mathematics content a very close fourth. Ranked fifth and clearly the lowest priority was development of teaching materials.

Table 80
Teacher Education

In what order of priority should the following areas within teacher education be addressed during the 1980's?

Item		*Professional*	*Lay*	*Total*
Mathematics content	High	17.3	12.4	16.2
	Low	40.8	27.0	37.6
Methods for teaching	High	33.2	34.1	33.4
	Low	3.8	5.7	4.2
Development of materials	High	5.6	7.9	6.1
	Low	29.7	27.0	29.1
Sensitivity to student needs	High	28.2	46.5	32.4
	High	7.5	7.0	7.4
Diagnostic and remediation strategies	High	16.8	11.6	15.6
	Low	15.5	29.1	18.6

UPGRADING ACTIVITIES

When requested to make further comparisons about the importance to be attached to upgrading activities during the 1980's, the rankings were the same for both lay and professional groups:

- improvement of methods and techniques for teaching mathematics
- improved pre-service and in-service education for teaching mathematics
- development of special mathematics materials for students with special needs
- development of non-text materials for teaching mathematics
- improved mathematics content for textbooks

Improved methods and techniques was clearly the first choice, receiving highest or second highest priority by seventy-four percent of the lay and sixty percent of the professional respondents.

Of those who ranked methods and techniques highest, sixty-eight percent indicated that they believed improvement in this area would have a far-reaching impact on mathematics education generally. An even higher percentage (seventy-six percent) of those who ranked improved pre-service and in-service as the highest priority also selected this reason.

Of the forty-four percent of the total sample who ranked improved mathematics content for textbooks as the lowest priority, fifty percent of them indicated they did so because the materials we presently have are more nearly sufficient. This reason was also given by those who rated the development of non-text materials as the lowest priority.

Table 81
Upgrading Activities

In previous questions you have ranked priorities within the broad areas of mathematics content, students with special needs and teacher education. To these areas might also be added the development of non-text teaching materials and the development of special teaching methods. In what order should these areas be studied or developed during the 1980's?

Item		*Professional*	*Lay*	*Total*
Mathematics content	High	13.6	13.1	13.5
	Low	49.5	25.0	44.1
Special materials for special students	High	20.8	19.3	20.5
	Low	11.6	9.6	11.2
Improved pre-service and in-service education	High	27.6	25.6	27.2
	Low	11.8	19.8	13.6
Development of non-text materials	High	10.9	6.0	9.8
	Low	17.4	38.6	22.1
Improvement of methods and techniques for teaching	High	27.4	41.7	30.6
	Low	6.8	2.4	5.9

GENERAL PROBLEMS

Respondents were asked to indicate the degree of seriousness of fifteen general problems facing all teachers. There was high agreement between lay persons and professionals on the rankings accorded these problems.

Ranked as highest, or most serious, were:

- reading difficulties
- unmotivated students

These were clearly the two most serious problems. Next were:

- no commitment to homework on the part of students or parents
- lowering of school academic standards
- increasing class size

The middle five problems, in order of seriousness, were:

- classroom discipline
- increased teacher workloads
- irregular attendance of students
- emphasis on non-academic school
- mixing of students with differing abilities in the same classroom

Those rated as least serious were:

- restrictions on instructional materials
- lack of community (this item had a word omitted which undoubtedly affected the responses)
- governmental or administrative restrictions
- too much free time for students
- decline in student abilities

Table 82
General Problems

Many general problems face teachers of mathematics (as well as other teachers). Indicate the seriousness of each of these problems:

Item		*Total*	*Professional*	*Lay*
Discipline	High	18.4	13.5	34.5
	Low	18.4	22.2	5.7
Lowering standards	High	17.2	14.4	26.1
	Low	5.9	5.6	6.8

Irregular attendance	High	9.7	9.8	9.4
	Low	17.8	19.6	11.8
Government or administrative restrictions	High	5.9	6.3	4.6
	Low	33.1	35.1	26.4
Lack of community	High	5.1	3.9	9.4
	Low	22.8	23.6	20.0
Reading difficulties	High	32.7	29.7	42.7
	Low	1.8	1.7	2.2
Unmotivated students	High	30.7	30.0	33.0
	Low	2.4	2.8	1.1
No commitment to homework	High	20.7	20.7	20.9
	Low	6.6	6.9	5.8
Decline in student abilities	High	3.2	2.8	4.6
	Low	35.5	34.5	39.1
Mixing students of different abilities	High	7.7	6.6	11.5
	Low	20.5	19.8	23.0
Increasing class size	High	18.4	20.8	10.6
	Low	15.0	11.4	27.1
Too much free time for students	High	5.0	2.4	14.0
	Low	27.3	29.6	19.8
Increased teacher workload	High	12.6	15.7	2.3
	Low	11.8	8.4	23.0
Emphasis on non-academic school	High	7.4	5.5	13.8
	Low	19.6	18.6	23.0
Restrictions on instructional materials	High	5.9	6.9	2.4
	Low	26.5	26.3	27.1

Respondents were then asked to indicate whether or not the general problems facing teachers deserve priority over those problems specific to the learning and teaching of mathematics. The general problems were clearly seen as deserving priority by lay persons and professionals alike.

DISTRIBUTION OF RESEARCH FUNDS

The responses about how research funds should be distributed in the 1980's revealed a first priority for investigations of how students learn, with forty-one percent agreeing. The second and third priorities were ranked well below the first, but close to one another; teaching methods, with twenty-four percent of the total sample, and teacher education with twenty-two percent. Low priority was assigned by both groups to varying types of materials and to longitudinal assessments of achievement.

Table 83
Research Funds

In your opinion, how should research funds be distributed among the following areas during the 1980's?

Item		*Professional*	*Lay*	*Total*
How students learn	High	43.3	33.7	41.1
	Low	10.7	4.7	9.3
Longitudinal assessments	High	6.2	5.7	6.1
	Low	39.5	34.1	38.3
Teaching methods	High	19.0	39.8	23.9
	Low	2.1	0.0	1.6
Teacher education	High	22.3	21.2	22.1
	Low	9.6	14.1	10.6
Varying types of materials	High	12.1	2.3	9.8
	Low	35.2	41.4	36.6

METHODS OF ATTACKING PROBLEMS

In assessing various methods of attacking problems in mathematics education, the rank ordering by the total group was:

. support the in-service education of teachers
. support evaluation of mathematics learning and achievement
. create a project to develop innovative teaching methods
. support pre-service education of teachers
. create many small, basic research projects
. give grants to local schools to improve their mathematics programs
. fund small, local curriculum development projects
. establish a few coordinated, long-term research projects
. establish a mathematics education clearinghouse for the collection of innovative materials
. fund research for study of general classroom problems
. support the development of non-text materials
. create a large curriculum development project with a nation-wide influence
. give grants to individual teachers for development of materials
. fund professional mathematics education organizations to publicize innovative ideas
. give grants to commercial firms for publishing innovative curriculum materials

Table 84
Methods of Attacking Problems

How do the following methods for attacking problems in mathematics education compare in general importance, practicality and efficiency?

Item		*Professional*	*Lay*	*Total*
Create many small, basic research projects	High	22.1	18.4	21.2
	Low	3.4	2.3	3.2
A few coordinated, long-term research	High	24.7	5.7	20.4
projects	Low	3.8	4.6	4.0
Small local curriculum development projects	High	24.4	15.1	22.3
	Low	3.8	5.8	4.3
Large curriculum development project	High	16.8	20.9	17.8
	Low	11.6	9.3	11.1
Grants to commercial firms for innovative	High	5.2	3.5	4.8
materials	Low	35.1	18.6	31.3
Grants to local schools	High	30.4	25.3	29.3
	Low	4.8	4.6	4.8
In-service of teachers	High	48.6	35.6	45.6
	Low	1.0	0.0	0.8
Evaluation of learning and achievement	High	23.0	41.9	27.3
	Low	1.4	2.3	1.6
Project for innovative methods	High	23.4	25.6	23.9
	Low	4.1	1.2	3.5
Development of non-text materials	High	19.7	9.5	17.4
	Low	5.5	2.4	4.8
Research general class problems	High	19.9	18.6	19.6
	Low	5.5	4.7	5.3
Mathematics education organizations to	High	15.0	9.3	13.7
publish innovative ideas	Low	5.5	11.6	6.9
Mathematics education clearinghouses	High	15.8	10.6	14.6
	Low	4.5	3.5	4.3
Grants to individual teachers	High	16.0	14.9	15.8
	Low	7.2	13.8	8.7
Support pre-service education	High	31.0	29.2	30.5
	Low	4.8	3.4	4.4

IV. IMPLICATIONS FOR ACTION

This final section of the report attempts to highlight in summary form the chief implications for action which appear to be supported by the PRISM–Canada data. These implications are intended to complement the interpretations and applications of the data which many readers will make for themselves. They may also serve to reinforce the relevance to the Canadian scene of a number of the recommendations outlined in *An Agenda For Action*.

Problem Solving Must be a Major Organizing Element in the Mathematics Curriculum.

Weaving problem solving experiences throughout the mathematics curriculum will develop methods of thinking and logical reasoning. To provide for this emphasis, there will need to be a reprogramming of the use of time in the classroom, more supplementary materials, greater use of calculators and computers and a shift in methodology.

Mathematics Programs Must Develop Computer Literacy.

Every student ought to be familiar with the role and impact of the computer and how to use it. This will require that they have access to computers and calculators in elementary and secondary school classrooms and that their use be integrated into the core mathematics curriculum.

Applications and Consumer Mathematics Warrant Greater Attention.

Applying mathematics to the real world, particularly in consumer and career situations, is fundamental to the development of problem solving ability and to student motivation and interest. Such realism calls for the use of techniques, activities and materials which go beyond the contents of most conventional textbooks.

The Diagnosis and Treatment of Learning Difficulties Must Be Addressed More Effectively.

Since mathematical competence is vital to every individual's meaningful and productive life, we must develop it in each student. Currently many students discontinue the study of mathematics at the earliest opportunity, often because of a lack of success. Concurrent with preventative provisions, individualized remediation efforts which correct deficiencies and build upon strengths need to be pursued at all levels of schooling.

Methods and Techniques for Teaching Mathematics Need Improvement.

Laymen and professionals agree that efforts to improve methods and techniques of teaching have the greatest potential for immediate and long-term upgrading in the quality of mathematics education. If this necessitates a re-direction of time, energy and money, then it probably could be at the expense of textbook revision and broad-scale assessment programs — at least in the short-run.

Preservice and Inservice Teacher Education Requires Revamping.

Significant modifications in both the pre-service and the in-service education of teachers must be undertaken to aid in the implementation of the changes in school mathematics envisioned in this report. These modifications must include incorporation of mathematical content, teaching methods and instructional technology materials designed to enable teachers to achieve new levels of performance in teaching problem solving, in using calculators and computers, as well as in diagnosing and treating learning difficulties.

Priority Ought to be Given in Research About How Students Learn.

Better understanding about how students learn mathematics is essential if we are to devise better ways of teaching them. While many significant benefits would accrue from applying in schools what we already know, much more knowledge about the learning of mathematics is critical to the development of a mathematically competent citizenry.

A CONCLUDING OBSERVATION

The identification of preferences and priorities for mathematics education in Canada during the current decade is a relatively easy task. Far more difficult — and far more important — is their interpretation into a sense of direction which can be translated into teaching-learning activities in classrooms.

This crucial endeavor demands of those in positions of influence in school systems, governments and universities leadership in stimulating, steering, and supporting the process. But it demands even more from teachers: a high and sustained level of motivation, professional competence and dedication.

APPENDIX

COMMENTS AND CRITICISMS

About 10 percent of the questionnaires in the survey were returned with comments. Because the writers generally were not writing for publication, but giving spontaneous reactions, it was felt that it would be unfair to them to include all the comments. However, a number of comments have been included to give some flavour to the criticisms and other concerns expressed by respondents.

The comments are organized into subcategories within two broad categories. The broad categories are comments about issues related to teaching and learning mathematics and comments about the study and the questionnaire.

I. Comments About Teaching and Learning Mathematics

Issues

Many of the items sparked comments on issues in mathematics education. Some examples follow:

Regarding homework, I recommend ½ hour in elementary school three times a week and ¼ hour in high school three times a week.

Some homework should be necessary but daily is too much. It turns kids sour on math (or any other subject for that matter).

A calculator should be used to:

a. explore mathematical relations effortlessly

b. reduce computational time.

All students should be allowed to use calculators once they have attained a good understanding of (and some proficiency in) basic math skills.

Whether or not a calculator should be used in taking tests or in doing practice exercises depends entirely on the objective of the test or exercise. Logic and the ability to test the "reasonableness" of one's answers are paramount in my view.

Work with computers — only after basic adding, subtracting, multiplication and division are automatic and fractions and decimals understood. The computer should be something of a reward for the bright students who will be able to use it correctly.

Work without computers — computers shouldn't be the last resort for lazy students. Early basic skills used to be mastered by grade five.

Learning by drill — every day for 15 minutes in elementary school.

The basics in math — tables, methods, etc. are absolutely crucial and regardless of how bored some students are, they must be learned. They must also be used enough so that the memory does not fail.

Content

Most of the following comments about the content of the mathematics program were made in response to specific stems:

Am I to assume that children should neither see nor hear about fractions with denominators greater than 12?

Common fractions often are more relevant and easier to grasp than decimals. "⅙ of a pie" is less abstract than "0.1666 of a pie".

"Less emphasis" on metric measurement only because it will automatically get the exposure needed when it is in essence the only measurement system in use.

The following comments also refer to the mathematics program but are more general in nature:

In high school (even elementary school for that matter) one should not teach probability. One should, however, introduce some elementary concepts which are probabilistic in nature. One should not teach number theory. One should, however, introduce some concepts which are number theoretical in nature. One should not teach logic. One should teach some elementary concepts that are drawn from the field of logic: etc., etc., etc.

The responses allowed me to emphasize the opinion that problem solving in mathematics should receive much more emphasis. I feel that the skill in problem solving would transfer to other academic programs. Many aspects of this skill are also applicable to general living skills.

When considering problem areas of the mathematics program, I feel preventative measures need emphasis. I consider a good knowledge of basic skills an essential part of mathematics. To encourage acquiring this knowledge, I suggest that Grade One students not be required to learn the number facts beyond "12". Including "facts to 18" appears too demanding for many Grade One students.

Integration of handicapped students is underway. I feel that provisions must be made for appropriate curriculum materials and qualified personnel to meet specific student needs at all levels, from Kindergarten to University.

Math should be fun and useful.

I'm aware that the world is changing — that knowledge that was considered essential yesterday is not necessarily so today — also that a solution must be found to the problem of how much we must learn to cope with tomorrow. The answers to those problems will not be forthcoming immediately. In the meantime, most people will be living and solving problems as they have always done. It is important that the average child not especially brilliant in math learn to deal with it.

Methods

Not many comments were received about methods used in teaching mathematics. The following four comments are representative of those received in this category.

A little time well spent is better by far than hours of non-productive work. I suspect that for the child who hates math, less time and more supervision during that time might help.

A lot of silly "finger fun" has confused children recently and has been quite a waste of time. Memorizing by drill is not a bad thing. It should be used in the classroom.

Teaching seems to be too compartmentalized, e.g., as long as the calculations are correct other matters such as neatness, writing, spelling, and English language of word mathematical problems are all but ignored.

One of my biggest beefs is the widespread spending of public monies on a myriad of projects whose outcome has little or no effect on its purpose. Our math educators within the universities must surely have sufficient contact with teachers at various levels and be keeping abreast of innovations through reading of their own professional journals, and such, to conduct most of this innovation and development of new material within the confines of their employ. And, hopefully, there will always be some parents and academics who are willing to volunteer their services.

Performance of Students

My observation is that schools do not give enough attention to the basics, hence high school graduates are all too often very poor readers, unable to do anything more than junior high mathematics — and that very laboriously.

Personally, I feel there are a lot of unmotivated students and there is declining support from parents BUT this is a whole new sociological issue.

It may seem that I'm not concerned with the extra bright child. I am, but good teaching plus the child's inquisitive mind will prepare him/her for the future. Public schools cannot be selective — they must simply provide a foundation — industry, governments, universities, science and technology must do the rest.

Again, I'd like to back our college-bound Grade twelves BUT — basing my opinion on local assesment, most of our general education Grade Twelves had not improved their knowledge of math from the Grade Eight level. In many guesses (and I mean 'guesses' literally), their math competence was lower than Grade Eight. These students need a great deal of help before we turn them loose on the work-a-day world.

II. Comments on the Survey

Many comments were received related to the study itself. These fall into two broad but related categories, comments about the items and the wording of the items, and comments about the reliability of the study.

Wording of the Questions

There were numerous comments about the wording of the questionnaires. Sometimes it was the strong wording of the choices or the interpreted action of the item which made response difficult. Other times the questions appeared "obscure" to the reader. The French surveys drew the most criticism in the language area. The language and language structure of the surveys in French were found lacking.

There were many who commented and found difficulty in responding to the statements in particular items. This led the respondents to give answers which they felt could be interpreted in many ways. Thus in considering "calculus" as unimportant, they were only responding that it was less important than other items. These persons were concerned with the interpretations which might be derived from such responses.

The questions were not nearly as well phrased in some cases as they should have been. "Ambiguous" may not be the proper description but "obscure" would certainly apply, as would "open to misinterpretation of intent".

Evidence for obscurity was the many comments on specific items that began with phrases or questions of the following form:

"It depends upon..."

"On what occasions...?"

"For what purposes...?"

"What age levels...?"

"Under what circumstances...?"

The following are two general comments received related to the questionnaire.

I dislike questionnaires of the type enclosed because there is no simple answer. There are always too many possibilities.

It would seem to me that more thought and planning could have gone into this questionnaire before asking people to give of their time to answer it.

Validity and Reliability

The first comment is specific but the others also reflect upon the validity of the study or reliability of the results.

I found this a poorly-worded and confused questionnaire, and I would question the validity of results coming from it.

I see very little evidence in the choice of items on this questionnaire that the researchers understand recent trends in mathematics education, developmental psychology, the political and social aspects of curriculum design, or typical teacher behaviour in the usual classroom context. I hope the other 75% of the items are better than these.

I apologize for finding myself unable to respond to questions dealing with expenditures of money for "resource materials". I am appalled at the preoccupation with this type of question in this survey; teaching (and learning) are not necessarily improved by spending money. What is needed is a consistent curriculum, books, and, on the part of the students, WORK; I assume good teachers are available. The attitudes reflected by this survey are most depressing.

> As is frequently the case with questionnaires, one is forced into extreme choices. I found that many of the questions forced me to deny the very kind of thoughtful professionalism I aim to develop in my own students. Without a knowledge of the content and the students these items are ludicrous. I have responded to them only to complete the questionnaire, not because I think that they are answerable as is.

Finally, there were persons who could not respond to the questionnaire because they questioned its philosophy and its vision. These persons, as did one critic above, questioned the forced choice questionnaire method for getting at priorities in school mathematics.

These critics commented that there were alternative visions of mathematics, teaching, learning and personal development which were not reflected in this survey. They felt that serious issues in the philosophy of mathematics education had not been addressed. They also suggested that this questionnaire looked at priorities much more in terms of "what is" rather than in presenting or finding out "what could best be".

No attempt will or perhaps could be made to respond to this block of criticism. It can be said that there were "holes" detected in this survey by numerous respondents. These criticisms also suggest that there are other models for assessing priorities in school mathematics. Hence, one must take care with the interpretation of the reported results and realize that these results represent only one view of the priorities in school mathematics.